Scientific Day in Memory of Prof. Mila Nikolova

My Work with Mila Nikolova

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Supported by HKRGC

Outline

1. **Impulse Noise Removal**
2. **SaT Segmentation Method**
3. **Hyperspectral Image**
4. **Rotating Point-spread Function**

Impulse Noise Removal

Salt-and-Pepper Noise

$\mathbf{f} = (f_{i,j})$: true image with $f_{i,j} \in [0, 255]$.

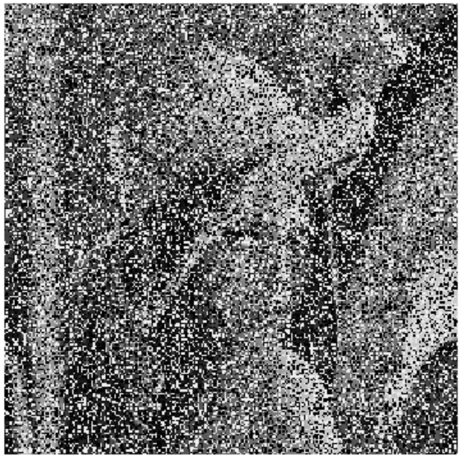
$\mathbf{y} = (y_{i,j})$: observed noisy image.

$$y_{i,j} = \begin{cases} 0 & \text{with probability } r/2\%, \\ 255 & \text{with probability } r/2\%, \\ f_{i,j} & \text{with probability } 1 - r\%. \end{cases}$$

Noise level $= r\%$.

$\mathbf{f}$

$\mathbf{y}$ with $r = 10\%$



$\mathbf{y}$ with $r = 50\%$

Median Filter

Noisy image

$y_{i-1,j-1}$	$y_{i-1,j}$	$y_{i-1,j+1}$
$y_{i,j-1}$	$y_{i,j}$	$y_{i,j+1}$
$y_{i+1,j-1}$	$y_{i+1,j}$	$y_{i+1,j+1}$

Restored image

$y_{i-1,j-1}$	$y_{i-1,j}$	$y_{i-1,j+1}$
$y_{i,j-1}$	y_{i_5}	$y_{i,j+1}$
$y_{i+1,j-1}$	$y_{i+1,j}$	$y_{i+1,j+1}$

sort

restore

$$y_{i_1} \leq y_{i_2} \leq y_{i_3} \leq y_{i_4} \leq y_{i_5} \leq y_{i_6} \leq y_{i_7} \leq y_{i_8} \leq y_{i_9}$$

median

30% Salt-and-Pepper Noise

Median filter

Every pixel is modified, hence fuzziness and blurring

Adaptive Median Filter

Noisy image

$$
\begin{array}{c|c|c}
y_{i-1,j-1} & y_{i-1,j} & y_{i-1,j+1} \\
\hline
y_{i,j-1} & y_{i,j} & y_{i,j+1} \\
\hline
y_{i+1,j-1} & y_{i+1,j} & y_{i+1,j+1}
\end{array}
$$

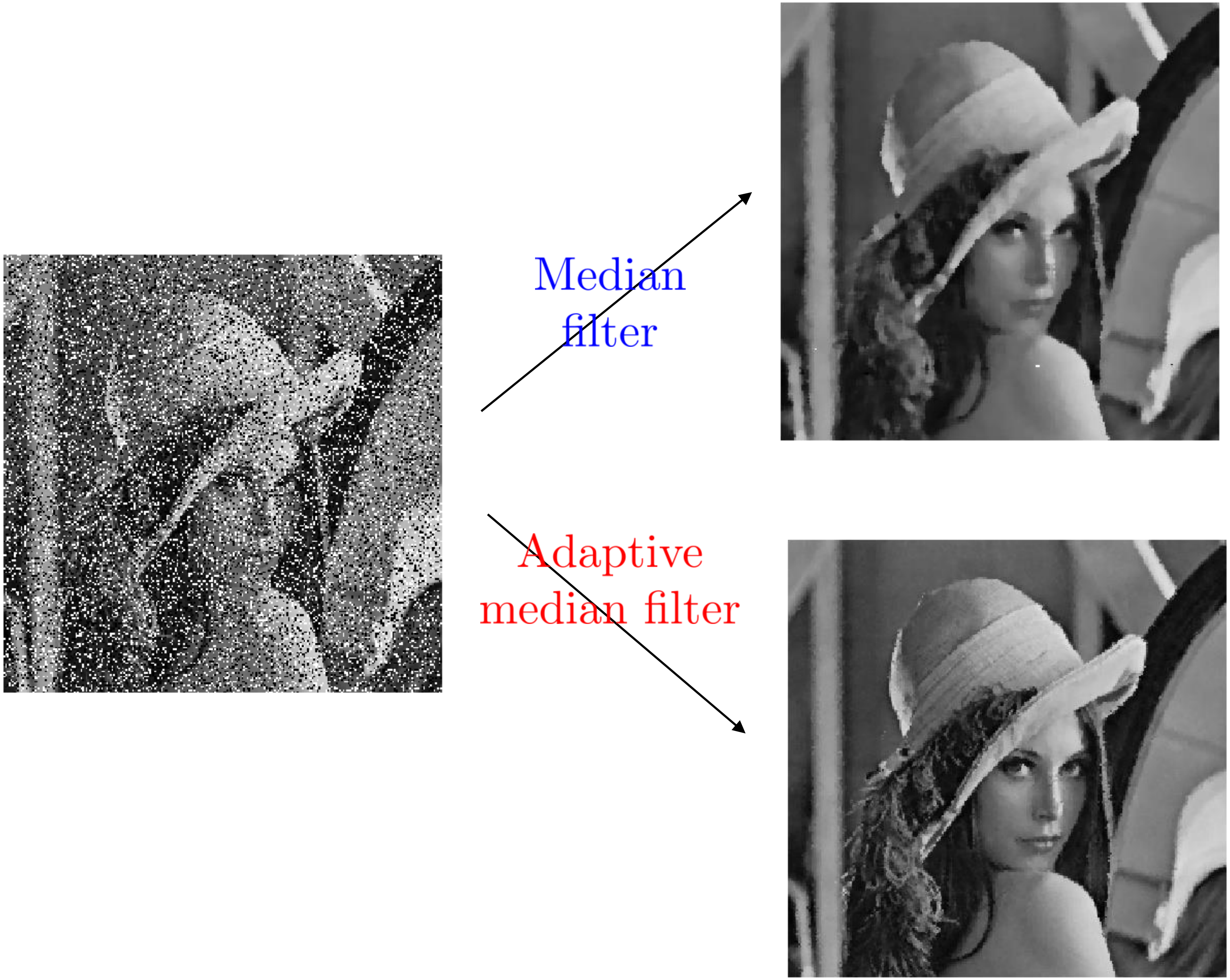
sort

$$y_{i_1} \leq y_{i_2} \leq y_{i_3} \leq y_{i_4} \leq y_{i_5} \leq y_{i_6} \leq y_{i_7} \leq y_{i_8} \leq y_{i_9}$$

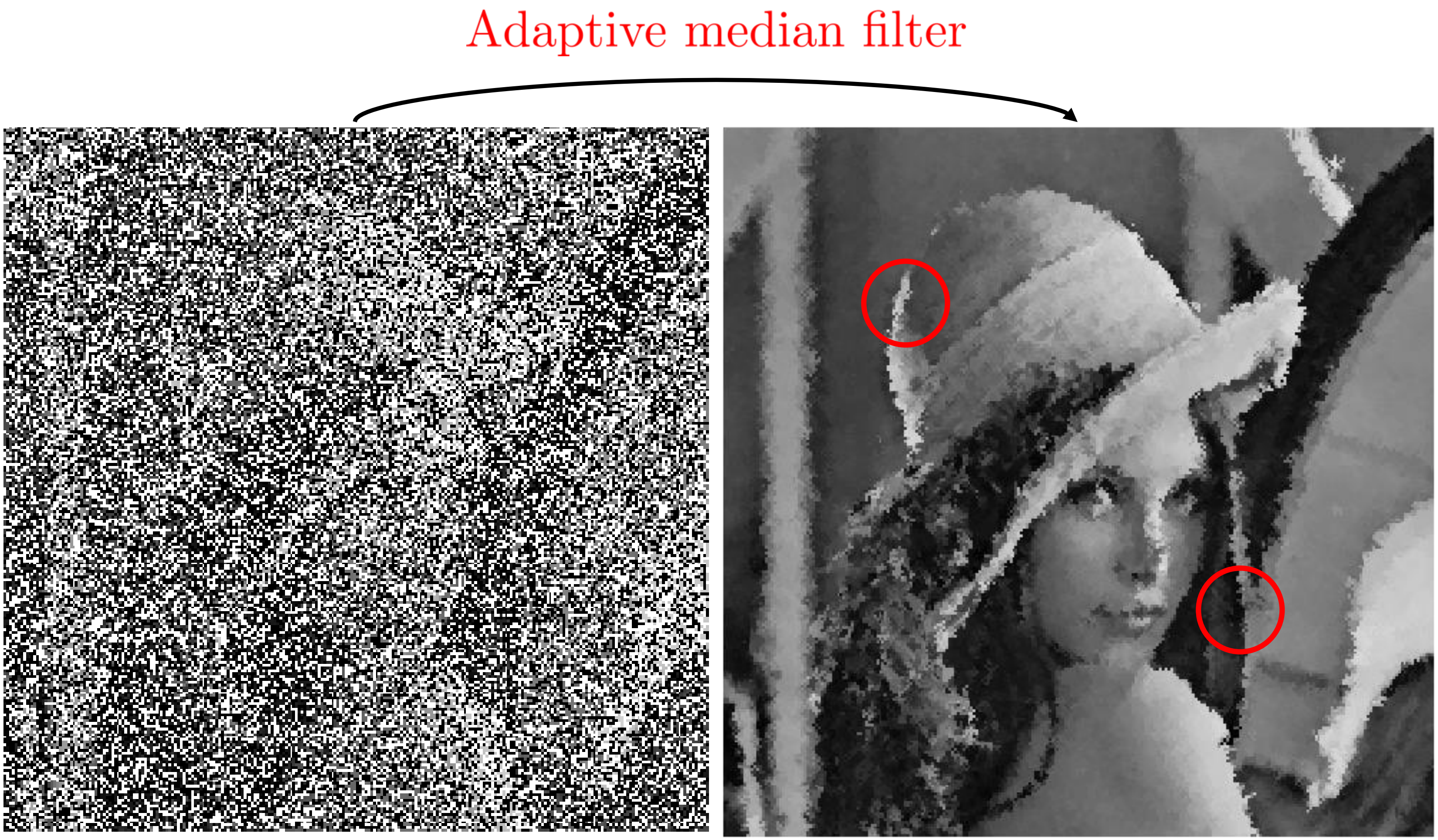
median

If median $= y_{i_1}$ or y_{i_9}, then increase window size.

30% Salt-and-Pepper Noise



But …at 70% Salt-and-Pepper Noise



Replacement of noise by median cannot preserve edges

l_1 Fitting Term for Impulse Noise

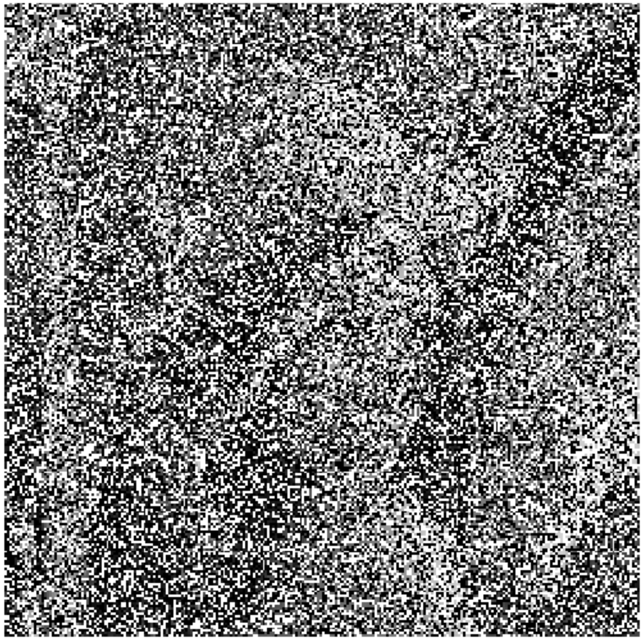
Nikolova, J. Math. Imaging & Vision, (2004)

$$F(\mathbf{f}) = \sum_{i,j} \Big[\overbrace{|f_{i,j} - y_{i,j}|}^{\ell_1\text{-norm data fitting term}} + \beta \overbrace{\sum_{(m,n)\in\mathcal{V}_{i,j}} \varphi_\alpha(f_{i,j} - f_{m,n})}^{\text{edge-preserving regularization term}} \Big]$$

- Non-smooth data-fitting term (smooth data left unchanged)
- Edge-preserving potential function:

$$\varphi_\alpha(t) = \begin{cases} |t|, & \text{total variation} \\ |t|^\alpha, & 1 < \alpha \le 2, \\ \sqrt{\alpha + t^2}, & \alpha > 0. \end{cases}$$

70% Salt-and-Pepper Noise



ℓ_1
model

Two-Phase Method

Chan, Ho, and Nikolova, IEEE TIP (2005)

Median-type Filter + Variational Method

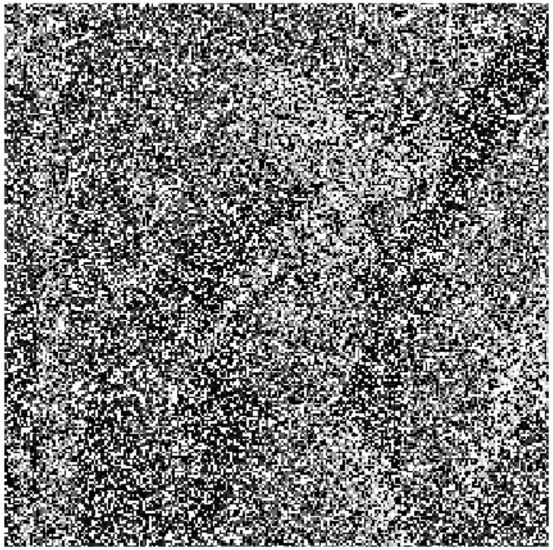
- Phase 1: Detect noise candidate set $\mathcal{N}$ by Adaptive Median Filter

- Phase 2: Restore pixels in $\mathcal{N}$ by ℓ_1 model

$$\begin{cases} \min\limits_{\mathbf{f}} \sum\limits_{i,j} \left[|f_{i,j} - y_{i,j}| + \beta \sum\limits_{(m,n) \in \mathcal{V}_{i,j}} \varphi_\alpha (f_{i,j} - f_{m,n}) \right], \\ \text{subject to } f_{i,j} = y_{i,j} \text{ if } (i,j) \notin \mathcal{N} \end{cases}$$

Solve the optimization problem on irregular grid-points $\mathcal{N}$.

Numerical Results



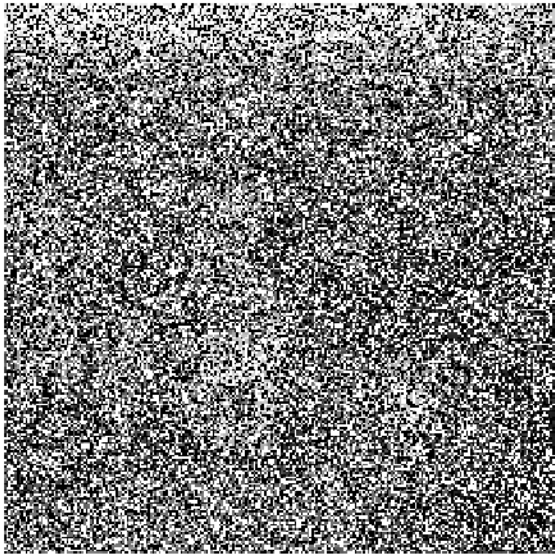
70% Salt-and-Pepper Noise

Adaptive Median Filter

Variational Method

AMF+Variation

Numerical Results



70% Salt-and-Pepper Noise

Adaptive Median Filter

Variational Method

AMF+Variation

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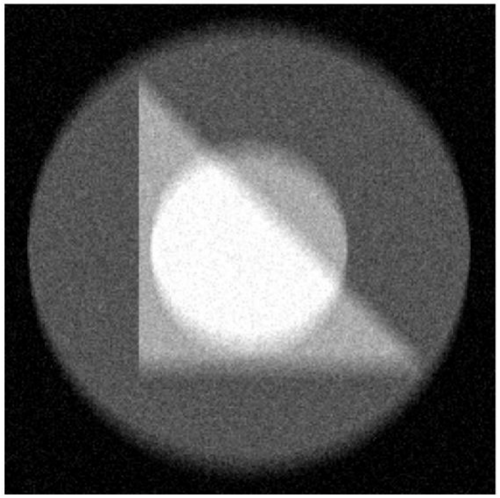
Segmentation: Problem Setting and Notation

Given a corrupted image f,

$f =$



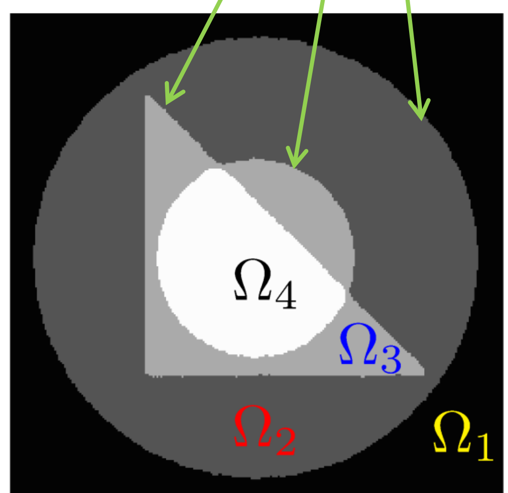
$= \Omega$

Γ

want a K-phase segmentation $=$

find a piecewise constant approximation with K constant regions

$g =$



$K = 4$

$$\Omega \setminus \Gamma = \cup_i \Omega_i,$$
$$g = c_i \text{ in } \Omega_i,$$
$$i = 1, 2, 3, 4$$

Mumford-Shah Model (1989) [cited 5,800+ time]

$f \xrightarrow{\text{minimize}}$ Mumford-Shah Energy $E_{\mathrm{MS}}(g, \Gamma)$

$$\Updownarrow$$

$$\frac{\lambda}{2}\int_{\Omega}(f-g)^2dx + \frac{\mu}{2}\int_{\Omega\setminus\Gamma}|\nabla g|^2dx + \mathrm{Length}(\Gamma)$$

Data fidelity: control g not far away from f	**Regularization:** impose smoothness of g on $\Omega \setminus \Gamma$	**Regularization:** require boundary Γ be short

Highly non-convex problem

Simplifying Mumford-Shah Model

Mumford-Shah Energy

$E_{\mathrm{MS}}(g, \Gamma)$

Simplify it:

$\nabla g \equiv 0$ on $\Omega \setminus \Gamma$

$\Updownarrow$

$= 0$

$$\frac{\lambda}{2} \int_\Omega (f - g)^2 dx + \cancel{\frac{\mu}{2} \int_{\Omega \setminus \Gamma} |\nabla g|^2 dx} + \mathrm{Length}(\Gamma)$$

Multiphase Chan-Vese Model (02)

(minimizer $\hat{g}$ is piecewise constant):

$$E_{\mathrm{MS}}(\{c_i\}, \Gamma) = \frac{\lambda}{2} \sum_{i=1}^{K} \int_{\Omega_i} (f - c_i)^2 + \mathrm{Length}(\Gamma)$$

Stage One: Convex Variant of the M-S Model

Restrict: $g \in W^{1,2}(\Omega)$ $\longrightarrow$ **Mumford-Shah Energy** $E_{\mathrm{MS}}(g,\Gamma)$

$$E_{\mathrm{MS}}(g,\Gamma) = \frac{\lambda}{2}\int_\Omega (f-g)^2 dx + \frac{\mu}{2}\int_{\Omega\setminus\Gamma} |\nabla g|^2 dx + \mathrm{Length}(\Gamma)$$

$m(\Gamma) = 0$

$\mathrm{Length}(\Gamma)$ $\Updownarrow$ approximated $\int_\Omega |\nabla g| dx$

$$\frac{\lambda}{2}\int_\Omega (f-g)^2 dx + \frac{\mu}{2}\int_\Omega |\nabla g|^2 dx + \int_\Omega |\nabla g| dx = E(g)$$

Convex M-S Energy $E(g)$

Two-Stage Segmentation Method

f → stage 1 → Convex M-S Energy

$$E(g)$$

$$\Updownarrow$$

$$\frac{\lambda}{2}\int_\Omega (f-g)^2dx + \frac{\mu}{2}\int_\Omega |\nabla g|^2 dx + \int_\Omega |\nabla g| dx$$

↓

smooth solution $\hat{g}$

↓

stage 2

↓

Threshold $\hat{g}$ to piecewise constant

Stage One: Extension to Blur/Projected Problems

$$\frac{\text{Convex M-S Energy}}{E(g)}$$

stage 1

$$\Updownarrow$$

$$\frac{\lambda}{2}\int_\Omega (f-g)^2dx \quad + \quad \frac{\mu}{2}\int_\Omega |\nabla g|^2dx \quad + \quad \int_\Omega |\nabla g|dx$$

$$\Big\downarrow \; f \leftarrow \mathcal{A}g + n$$

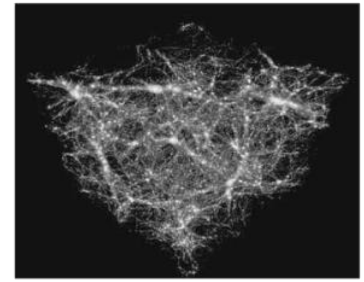
$$\frac{\lambda}{2}\int_\Omega (f-\mathcal{A}g)^2dx \quad + \quad \frac{\mu}{2}\int_\Omega |\nabla g|^2dx \quad + \quad \int_\Omega |\nabla g|dx$$

- Extendable to images corrupted by blur or projection $\mathcal{A}$
- Convex model with unique solution $\hat{g}$

Our Two-stage Segmentation Algorithm

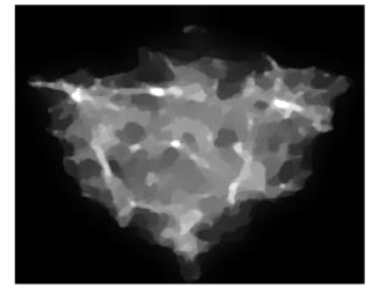
Only 1 convex problem to solve

Given f



Stage 1: solve $\hat{g}$ in

$$\min_g \left\{ \frac{\lambda}{2}\|f - \mathcal{A}g\|_2^2 + \frac{\mu}{2}\|\nabla g\|_2^2 + \|\nabla g\|_1 \right\}$$



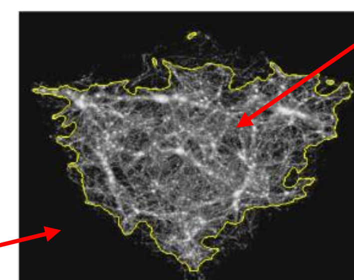
No iterations between Stages 1 and Stages 2

Stage 2: determine threshold ρ from $\hat{g}$ by mean or K-mean

ρ=0.19

K phases

$\hat{g} > 0.19$



$\hat{g} \leq 0.19$

Advantages of Smooth-&-threshold (SaT) Method

Given f

↓

Stage 1: solve $\hat{g}$ in

$$\min_g \left\{ \frac{\lambda}{2}\|f - \mathcal{A}g\|_2^2 + \frac{\mu}{2}\|\nabla g\|_2^2 + \|\nabla g\|_1 \right\}$$

↓

Stage 2: determine thresholds $\{\rho_i\}_{i=1}^{K-1}$ from $\hat{g}$

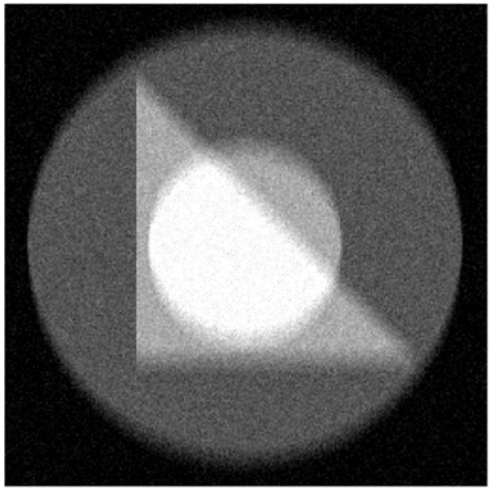
↓

K phases

Advantages

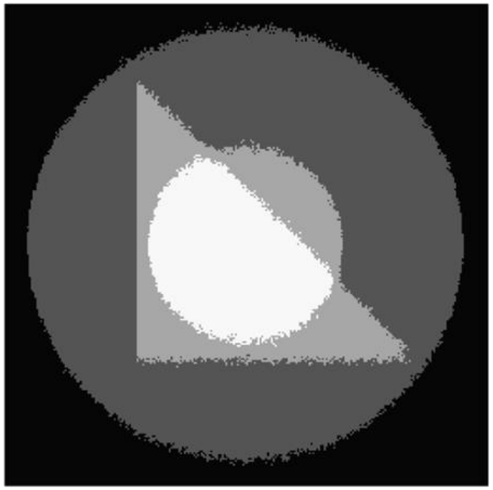
- Stage 1 model for finding $\hat{g}$ is convex
- Stage 2 uses the same $\hat{g}$ when thresholds ρ_i or K change (No need to recompute $\hat{g}$)
- No need to fix K at the very beginning
- Easily adapted to different kinds of corruptions (e.g. blur, projection, non-Gaussian noise)

4-phase Segmentation of Noisy and Blurry Image

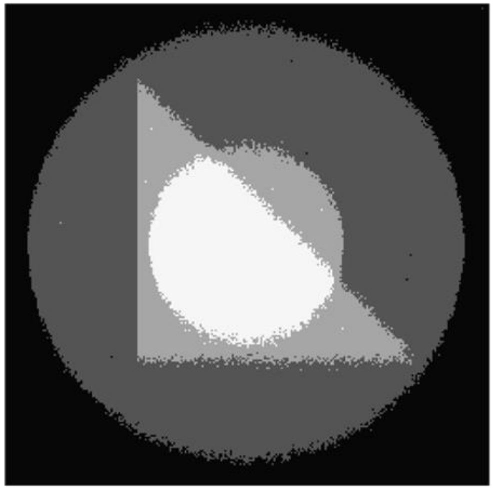


Noisy & blurry

Yuan *et al.* (10)

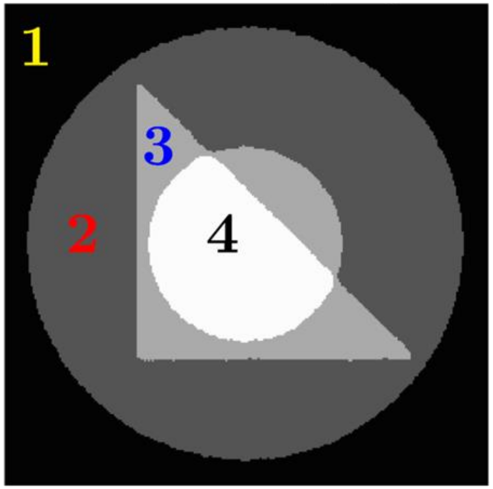


Li *et al.* (10)



Sandberg *et al.* (10)

Steidl *et al.* (12)



Our 4 phases from $\hat{g}$ using K-means ρ_i

Segmentation under Poisson or Gamma Noise

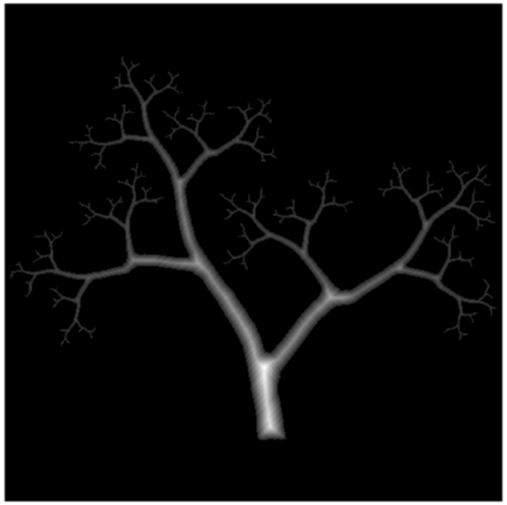
First stage: given f, solve

$$\min_{g}\left\{\lambda\int_{\Omega}(\mathcal{A}g - f\log\mathcal{A}g)dx + \frac{\mu}{2}\int_{\Omega}|\nabla g|^2 dx + \int_{\Omega}|\nabla g|dx\right\}.$$

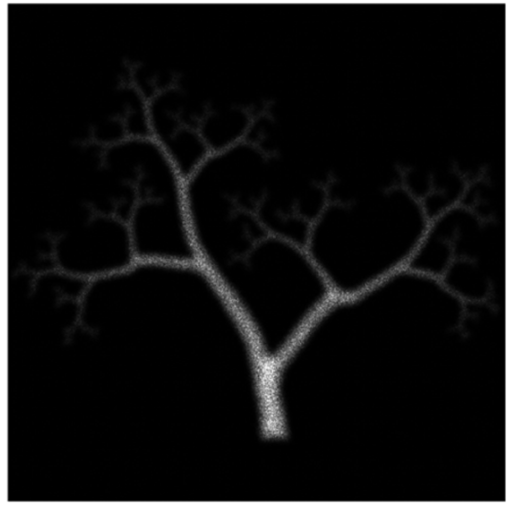
- ☐ data fitting term good for Poisson noise from MAP analysis
- ☐ also good for multiplicative Gamma noise (Steidl and Teuber (10))
- ☐ objective functional is convex (solved by Chambolle-Pock)
- ☐ admits unique solution $\hat{g}$ if $\mathrm{Ker}(\mathcal{A}) \cap \mathrm{Ker}(\nabla) = \{0\}$

Second stage: threshold the solution $\hat{g}$ to get the phases.

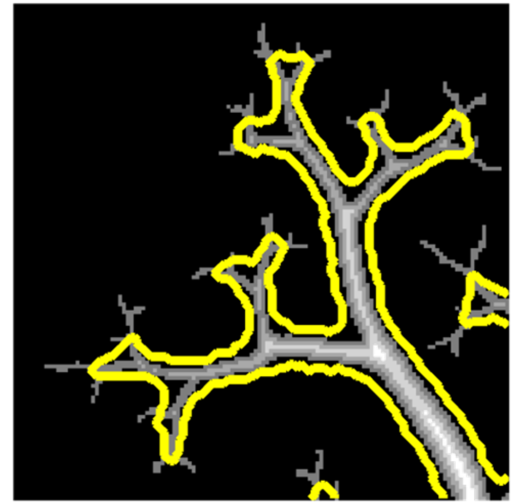
Fractal Tree with Gamma Noise and Gaussian Blur



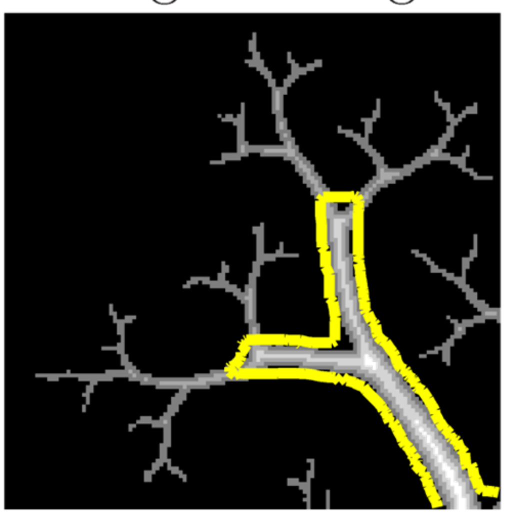
Original image



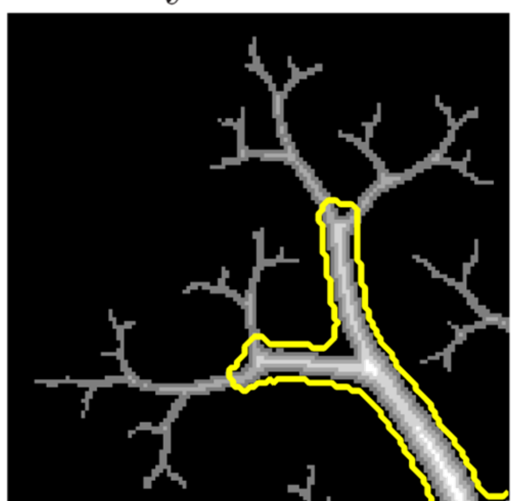
Noisy & blurred



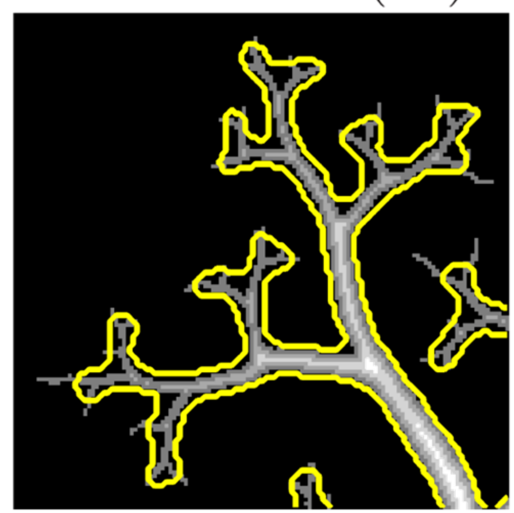
Yuan *et al.* (10)



Dong *et al.* (10)

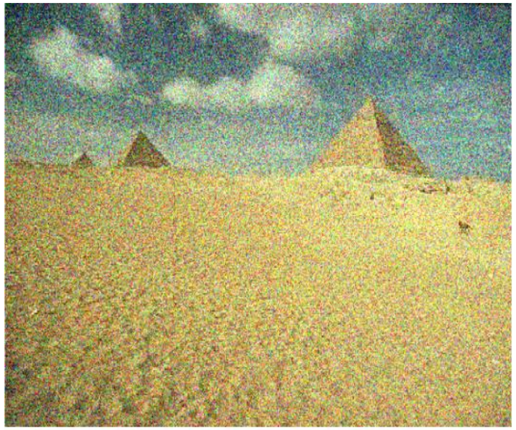


Sawatzky *et al.* (13)



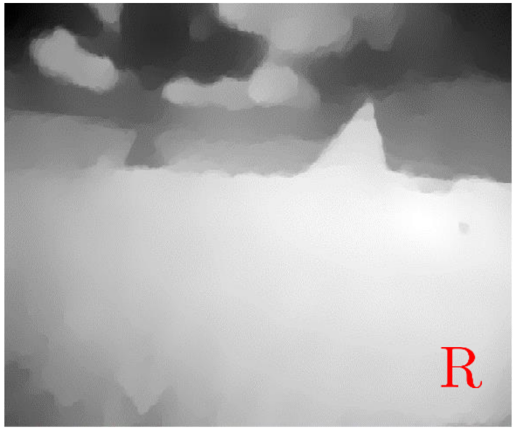
$\rho_{\text{user}} = 14$

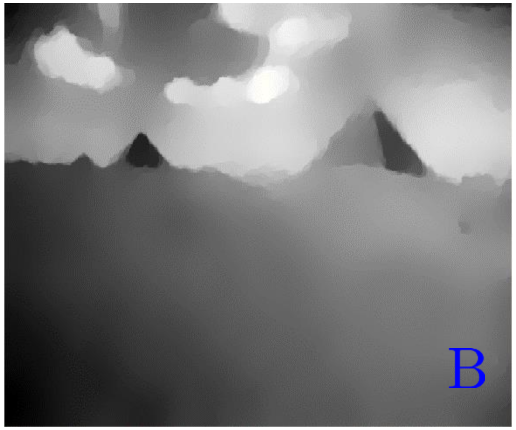
Is 2-stage Enough for Color Images?



noisy image

K-mean thresholding

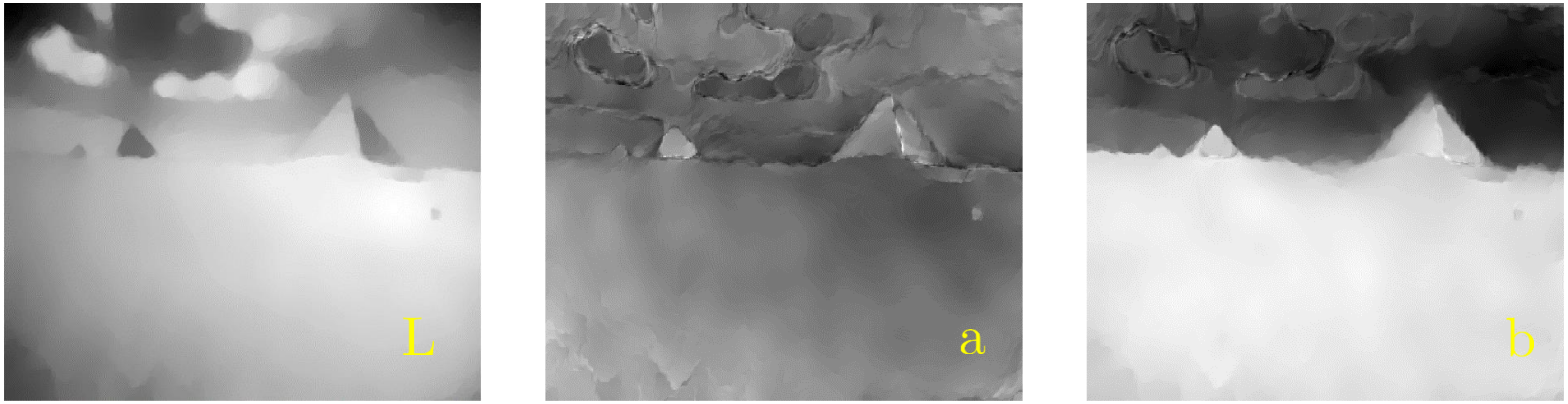




RGB: strong inter-channel correlation

Less-correlated Color Space

RGB: strong inter-channel correlation



Lab channels: less correlated

Thresholding using all six-channels

Three-stage (SLaT) *Method for Color Images*

Stage 1 (smoothing): given $f = (f_1, f_2, f_3)$, solve

$$\min_{g_i} \left\{ \lambda \int_\Omega (\mathcal{A} g_i - f_i)^2 dx + \frac{\mu}{2} \int_\Omega |\nabla g_i|^2 dx + \int_\Omega |\nabla g_i| dx \right\}, \ i = 1, 2, 3,$$

to obtain smooth unique solution $\hat{g} = (\hat{g}_1, \hat{g}_2, \hat{g}_3)$.

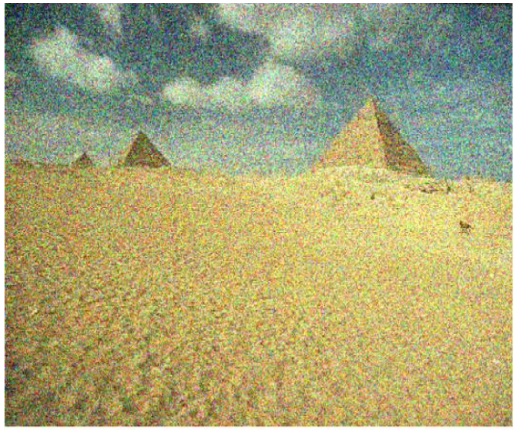
Stage 2 (lifting):

- transform $\hat{g}$ to another color space $\bar{g} = (\bar{g}_1, \bar{g}_2, \bar{g}_3)$ with less-correlation among the channels
- Then form the uplifted image $g = (\hat{g}_1, \hat{g}_2, \hat{g}_3, \bar{g}_1, \bar{g}_2, \bar{g}_3)$

Stage 3 (thresholding): Use K-means to threshold uplifted image g to get the phases.

2-phase Segmentation for Noisy Color Image

Clean image



Noisy image

Li *et al.* (10)

Pock *et al.* (09)

Strorath *et al.* (14)

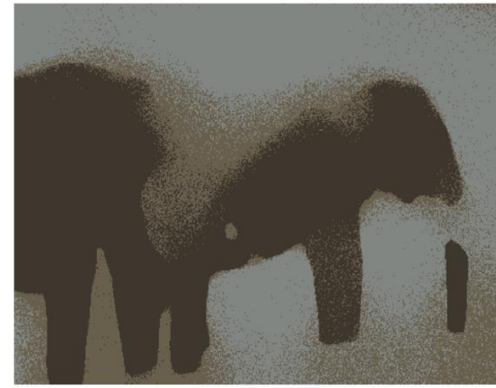
SaT with thresholds from K-means

Gaussian noise with s.d. 0.1.

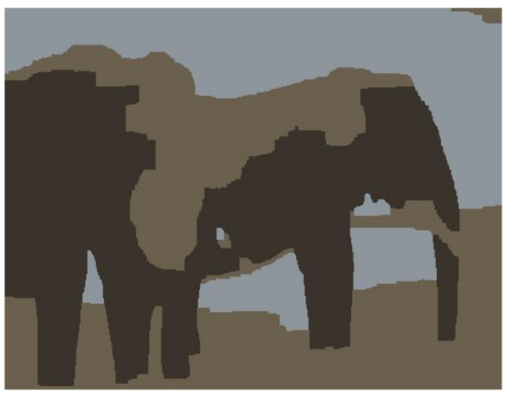
3-phase Segmentation for Noisy & Blurry Image

Clean image

blurry & noisy



Li *et al.* (10)



Pock *et al.* (09)

Strorath *et al.* (14)

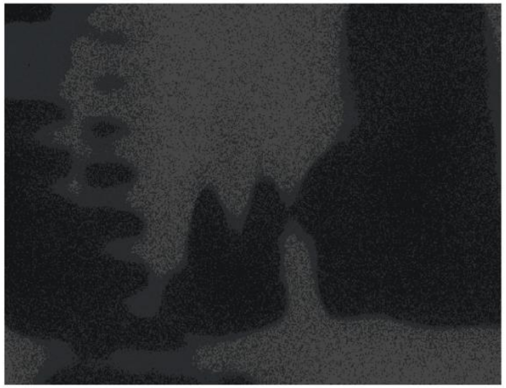
SaT with thresholds from K-means

10-pixel vertical motion blur with Poisson noise added

4-phase Segmentation for Pixel-loss Color Image

Clean image

Noisy image



Li *et al.* (10)

Pock *et al.* (09)

Strorath *et al.* (14)

SaT with $\rho_{K=4}$

60% pixel loss with Poisson noise added

Cai, C., Nikolova, and Zeng, J. Sci. Comput., (2017)

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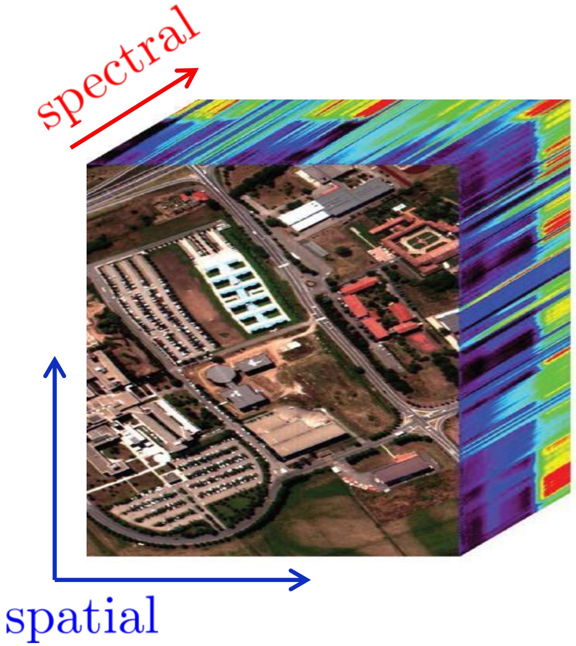
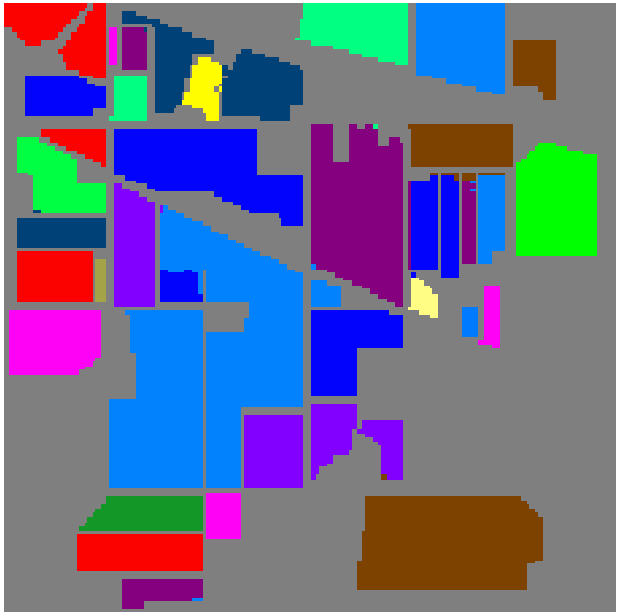
Hyper-spectral Image Classification

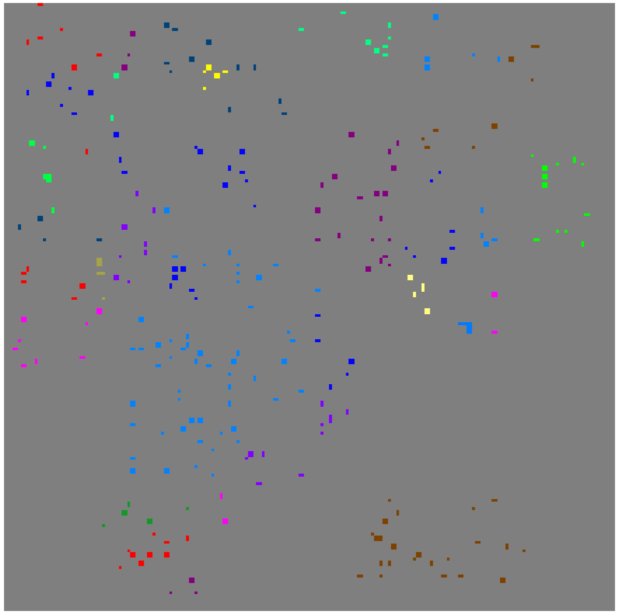
analyze the material for each pixel





Our method

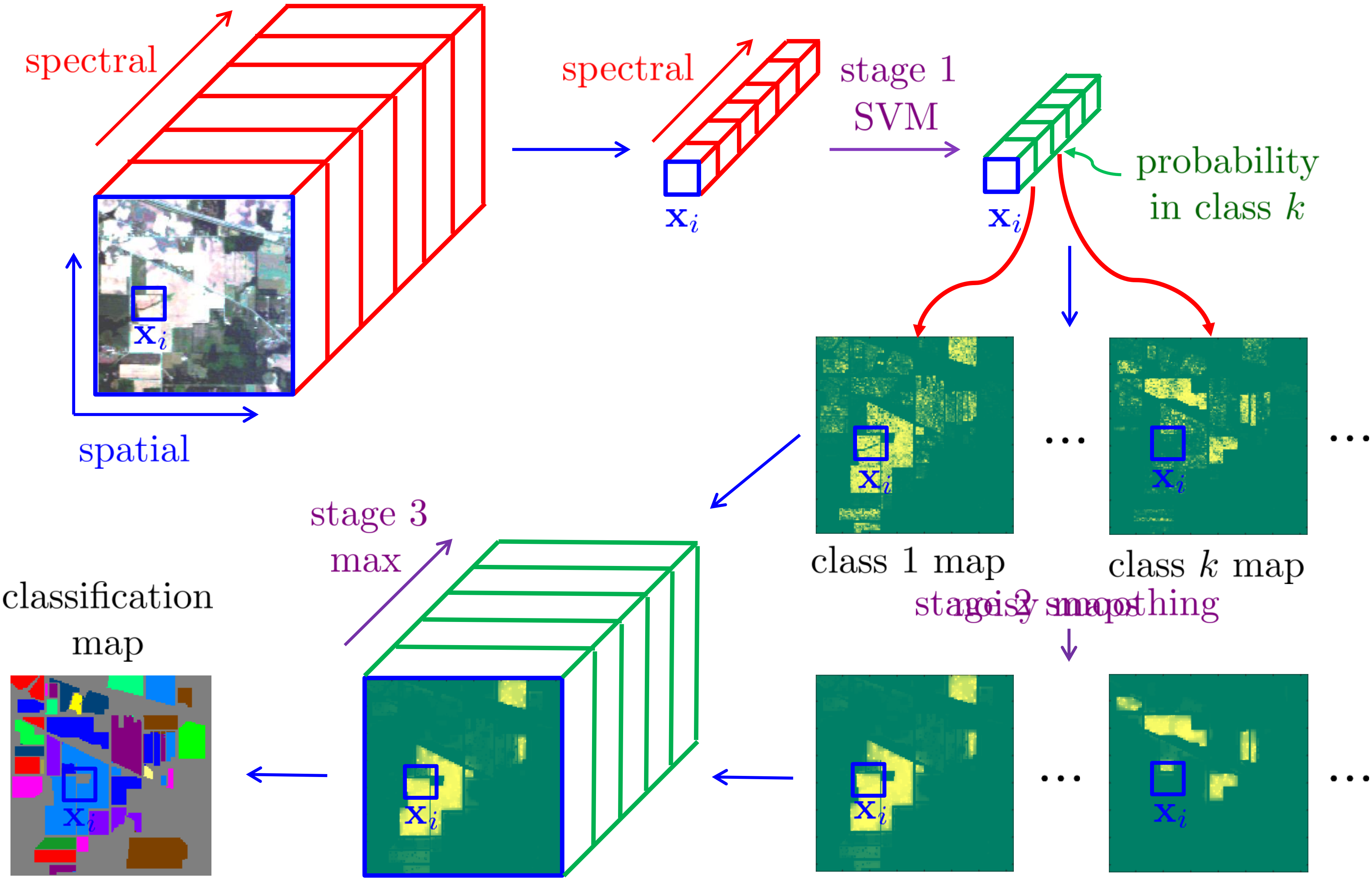
16 classes
of materials



Training pixels
10% = 1048 pixels

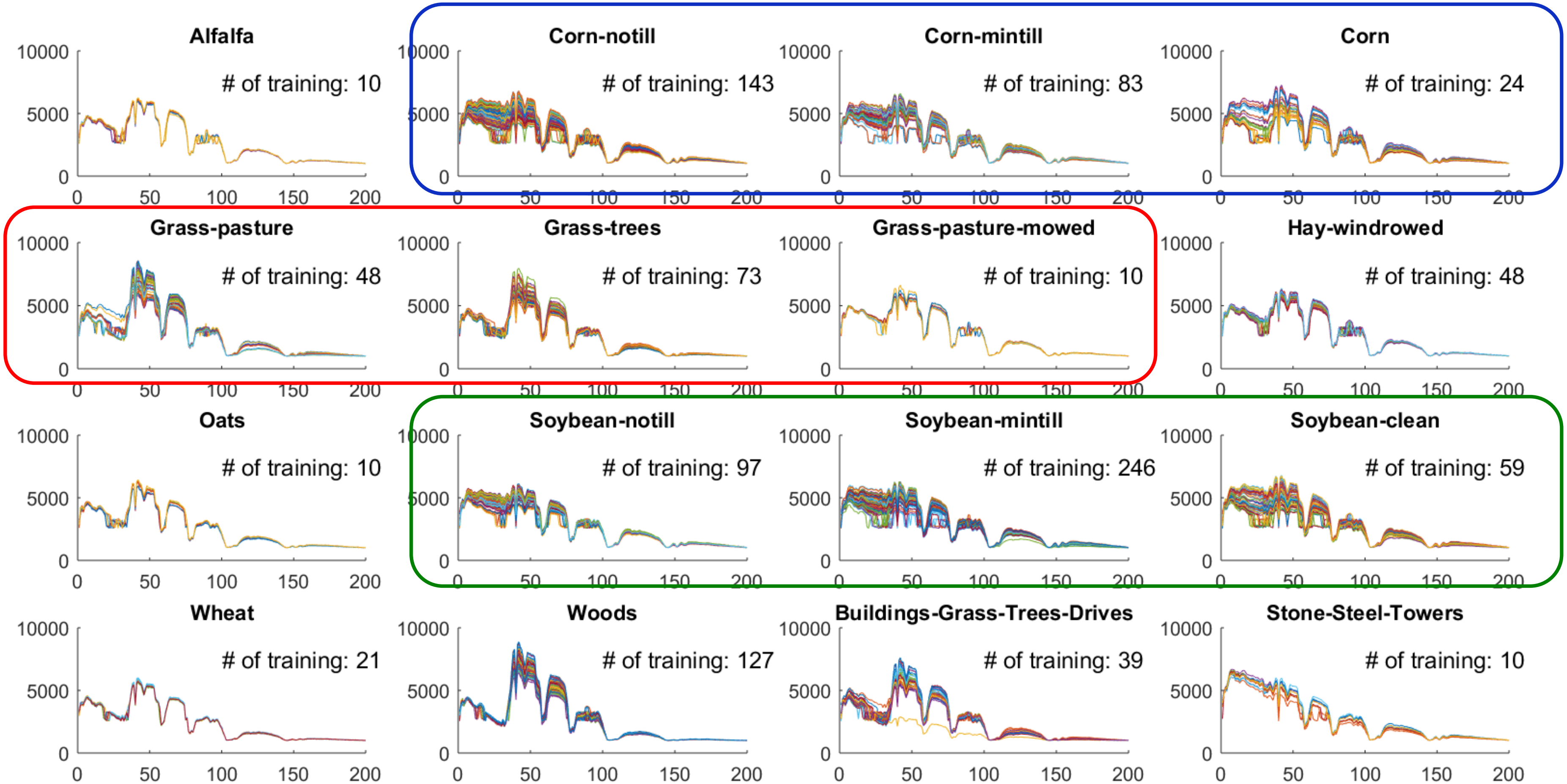
Our Smooth and Threshold (SaT) Approach

Traditional SVM-Method



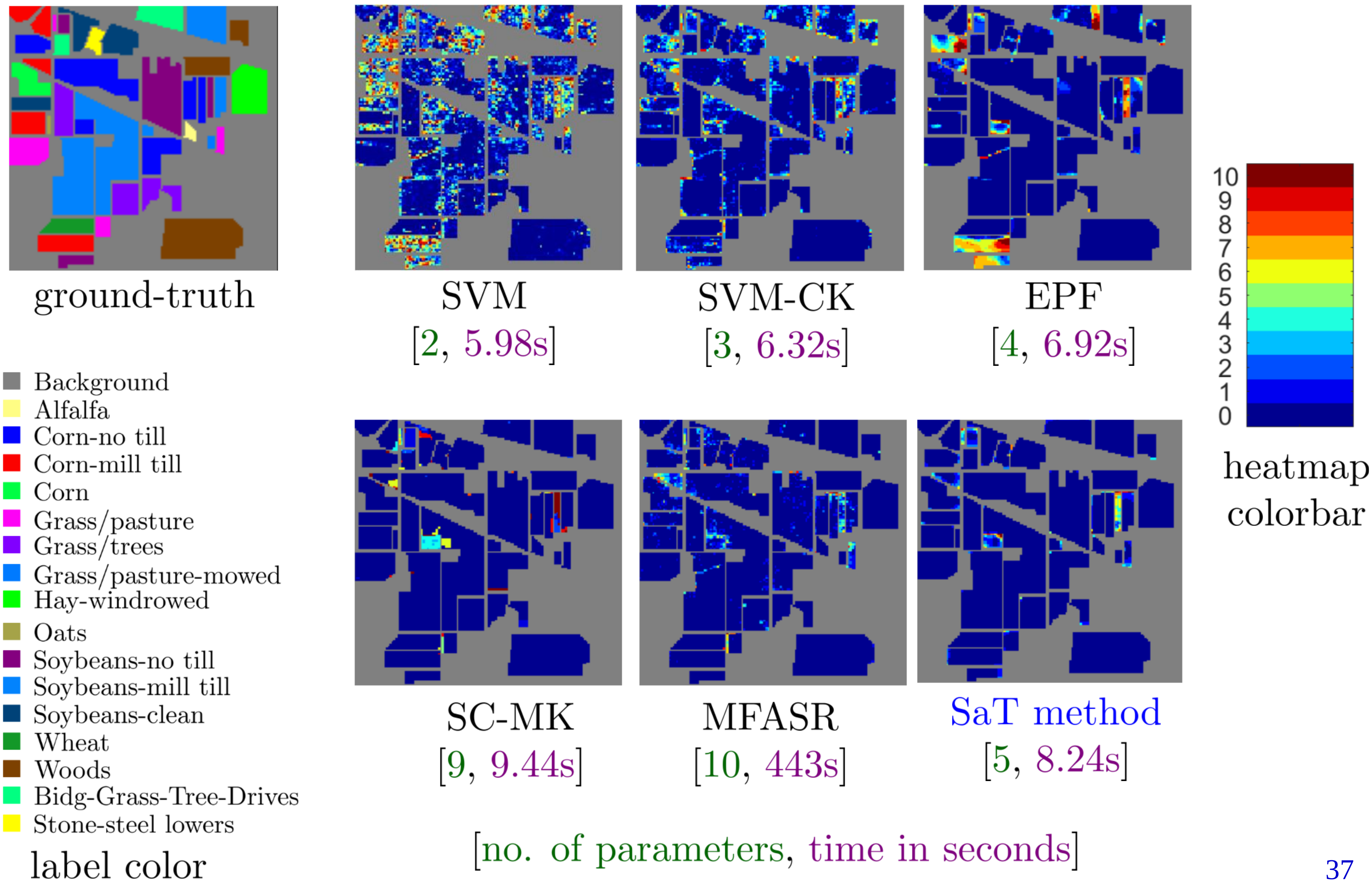
Indian Pines Data Set

- Data size: 145 × 145 (spatial) × 200 (spectral)
- Close spectrum between classes



Indian Pines Data Set

Error heat map over 10 trials with random 10% training pixels



Comparison with Other Methods

Accuracy over 10 random trials with random 10% training pixels

	SVM	SVM-CK	EPF	SC-MK	MFASR	SaT	gain
overall accuracy	79.78%	92.11%	93.34%	97.83%	97.88%	98.83%	0.95%
average accuracy	80.11%	92.68%	95.95%	98.35%	97.91%	98.88%	0.35%
kappa	76.90%	91.01%	92.36%	97.52%	97.58%	98.66%	1.08%

- ☐ overall accuracy: percentage of correctly classified pixels
- ☐ average accuracy: average of the accuracy in each class
- ☐ kappa: Cohen's kappa coefficient

SVM [Melgani *et al.*, 2004], SVM-CK [Camps-Valls *et al.*, 2006], EPF [Kang *et al.*, 2014], SC-MK [Fang *et al.*, 2015], MFASR [Fang *et al.*, 2017].

Effect of the High-order Smoothing Term

Accuracy over 10 random trials with random 10% training pixels

	SaT with $\|\nabla g\|^2$	SaT without $\|\nabla g\|^2$	gain
overall accuracy	98.83%	97.26%	1.57%
average accuracy	98.88%	95.89%	2.99%
kappa	98.66%	96.86%	1.80%

- ☐ overall accuracy: percentage of correctly classified pixels
- ☐ average accuracy: average of the accuracy in each class
- ☐ kappa: Cohen's kappa coefficient

C., Kan, Nikolova, and Plemmons, arXiv 1806.00836.

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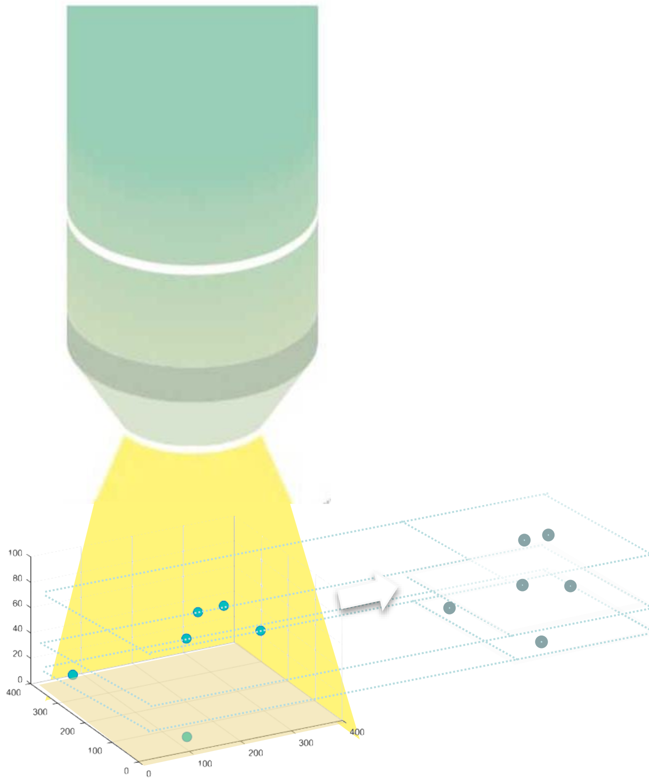
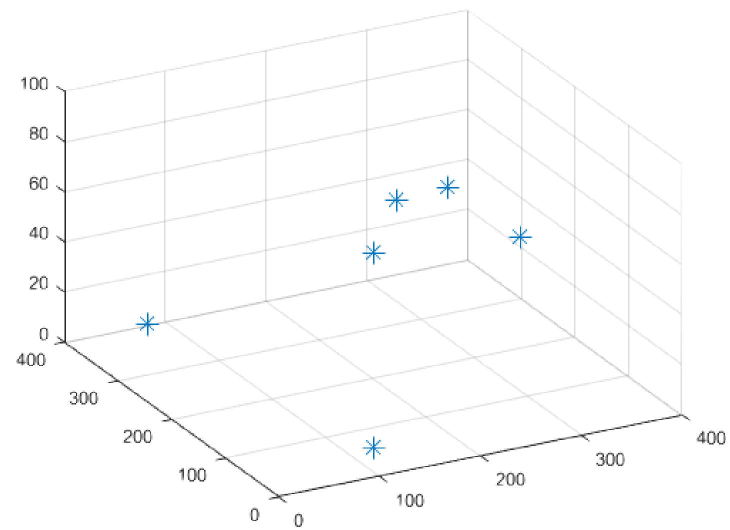
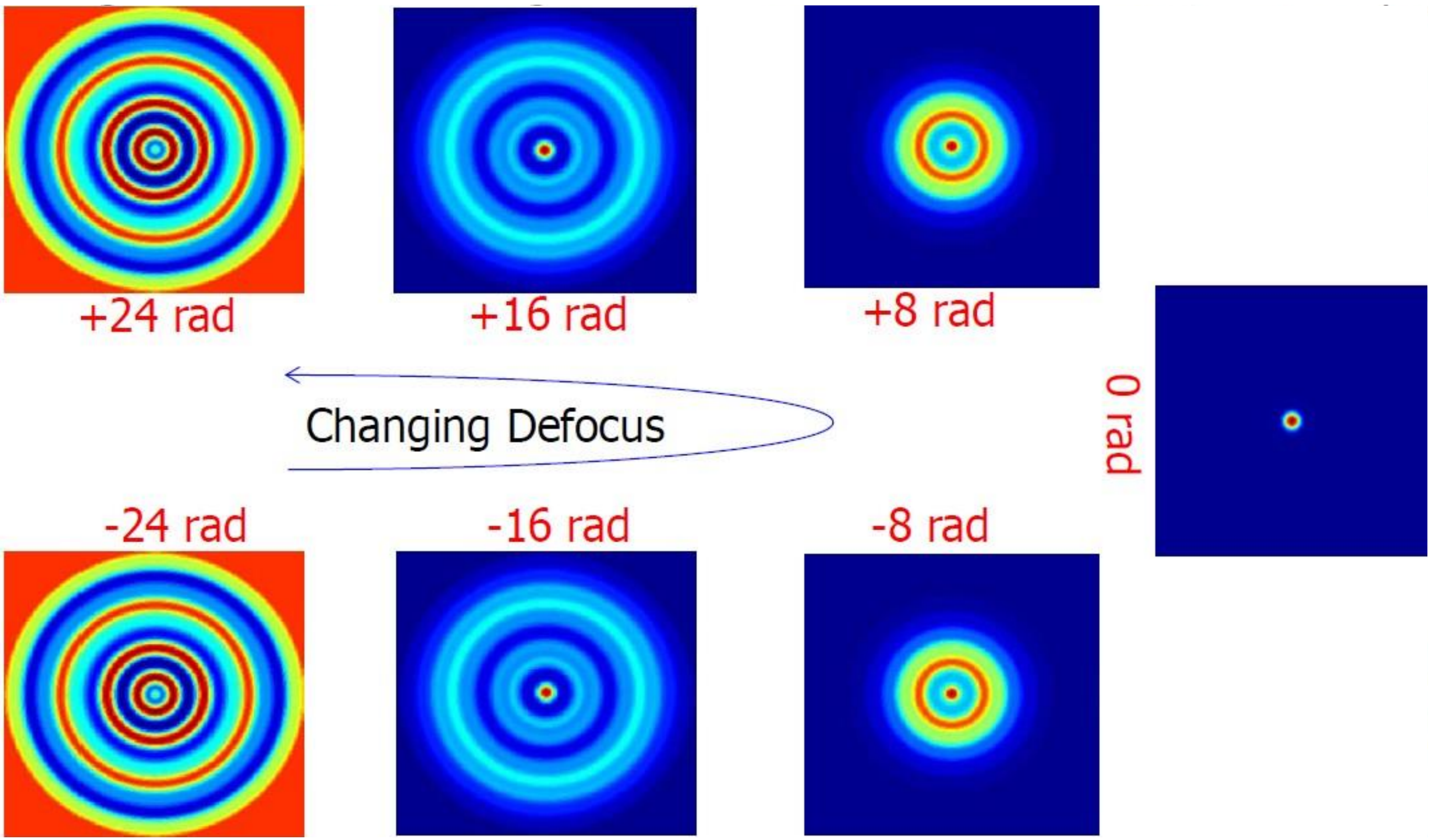
3D Imaging by 2D (Depth from Focus)

3D localization by scanning sequence of 2D images



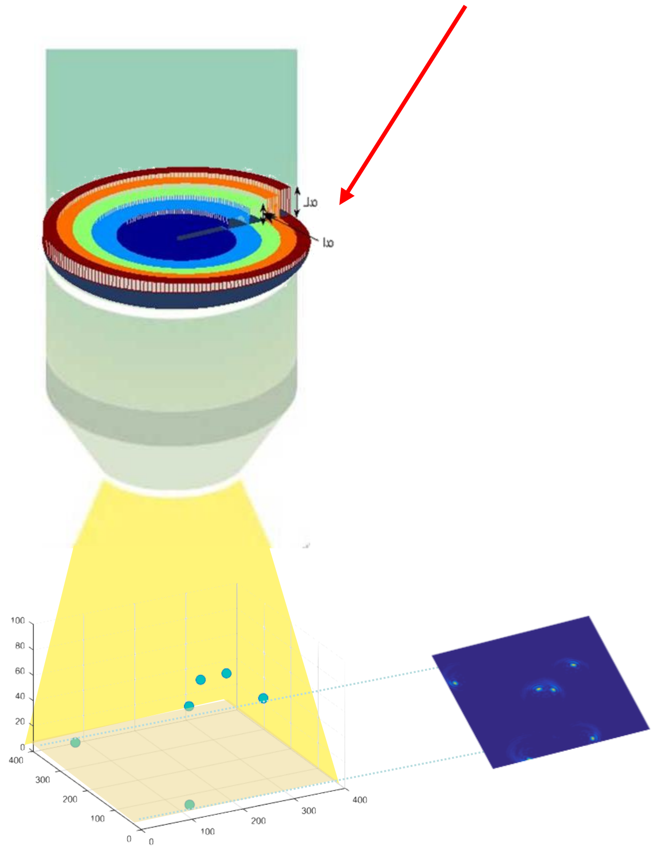


Conventional PSF for Single Point Source

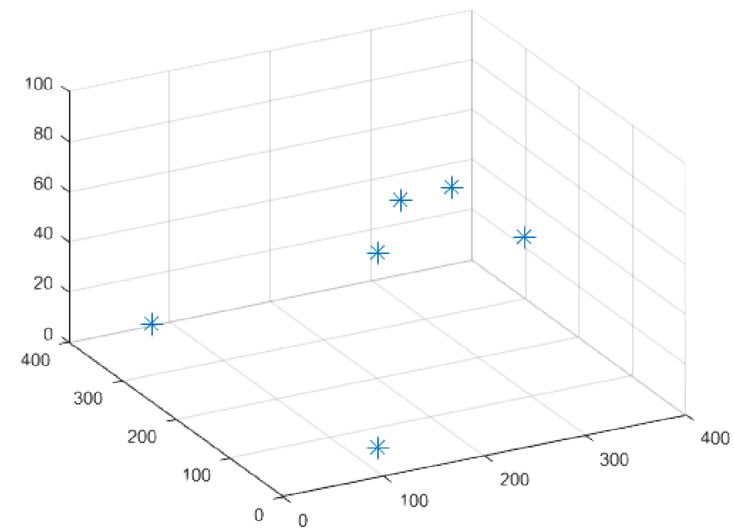


3D Imaging by 2D (Depth from De-focus)

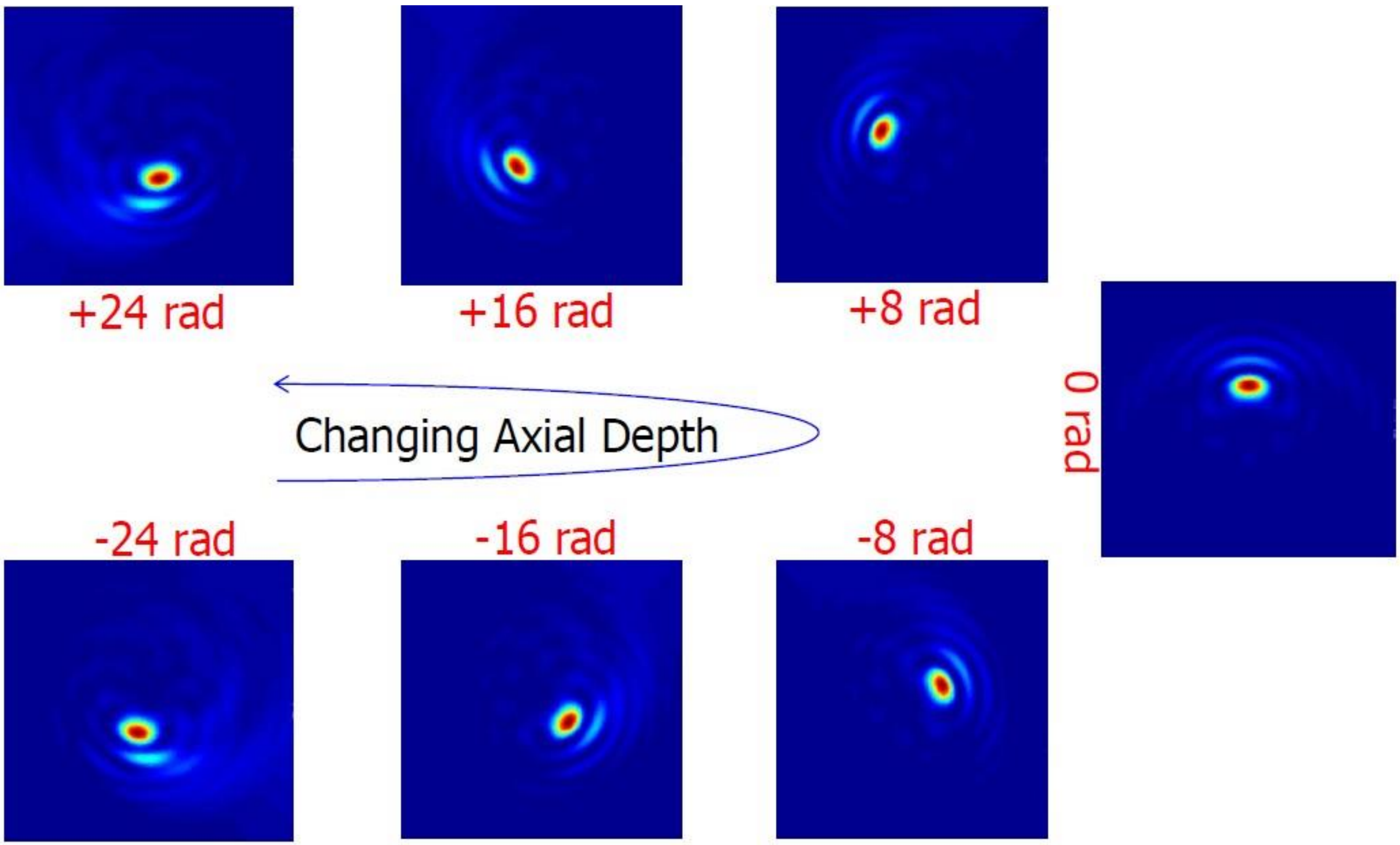
3D localization by rPSF



Engineer PSF to obtain
3D info from one 2D snapshot

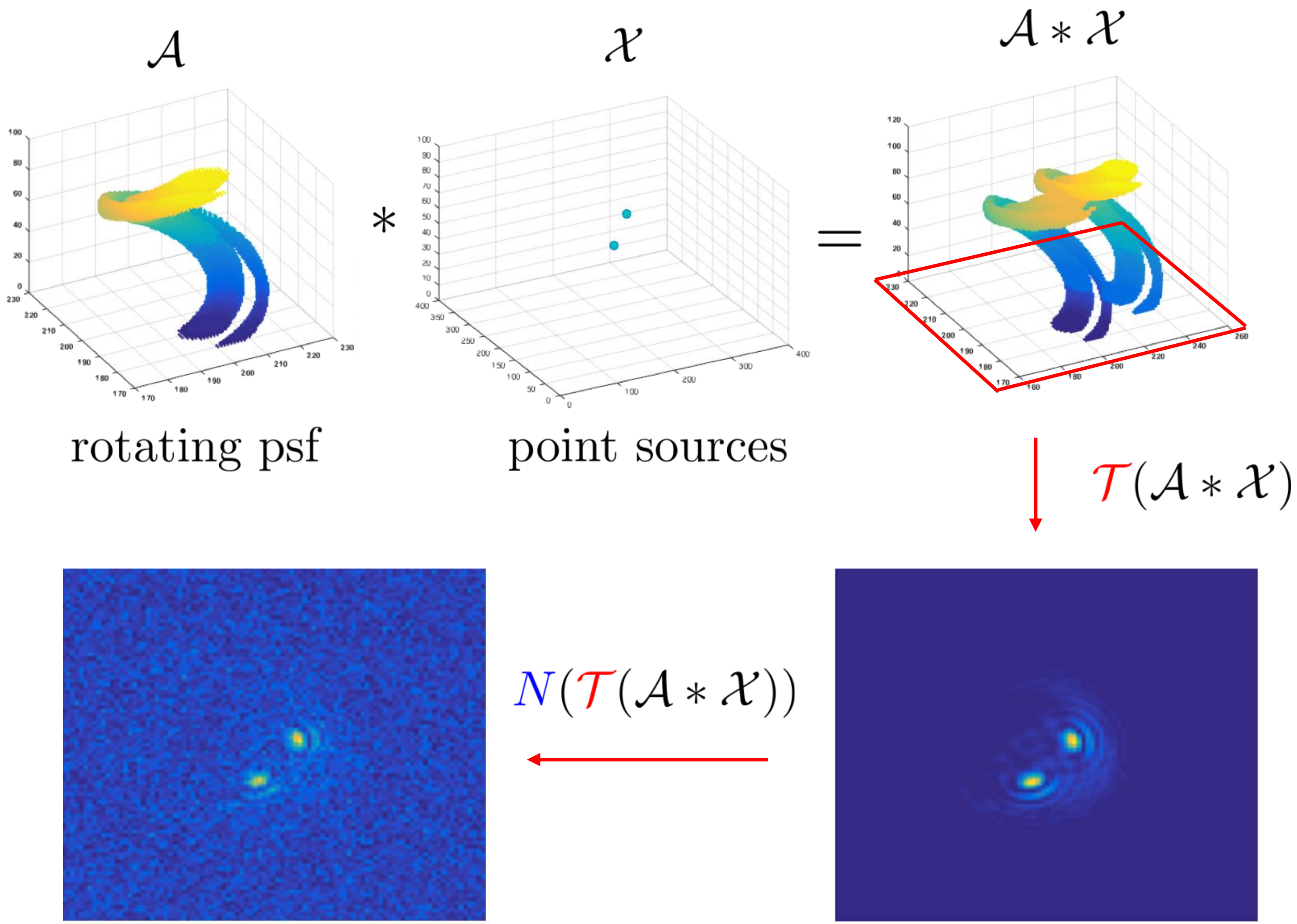


Single Lobe Rotating PSF



Forward Model for Rotating PSF

$$\boxed{G = N(\mathcal{T}(\mathcal{A} * \mathcal{X}))}$$

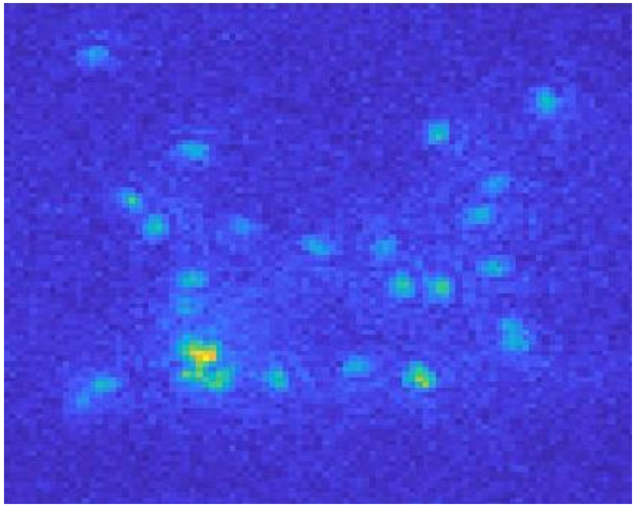


Kullback-Leibler + Nonconvex Model

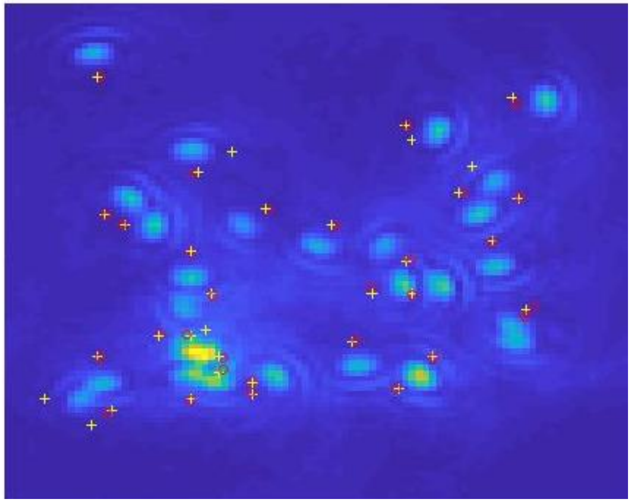
$$\min_{\mathcal{X}\geq 0}\left\{\underbrace{\langle 1, \mathcal{T}(\mathcal{A}*\mathcal{X}) - G\ln(\mathcal{T}(\mathcal{A}*\mathcal{X}) + b\,1)\rangle}_{\text{KL distance for Poisson noise}} + \underbrace{\mu\sum_{i,j,k=1}^{m,n,d}\frac{|\mathcal{X}_{ijk}|}{a+|\mathcal{X}_{ijk}|}}_{\text{non-covex regularizer}}\right\}$$

- ☐ solve by iterative reweighted ℓ_1 algorithm
- ☐ substitute $\mathcal{U}_0 = \mathcal{A}*\mathcal{X}$, $\mathcal{U}_1 = \mathcal{X}$ and solve by ADMM
- ☐ close-form solution for subproblem related to KL distance

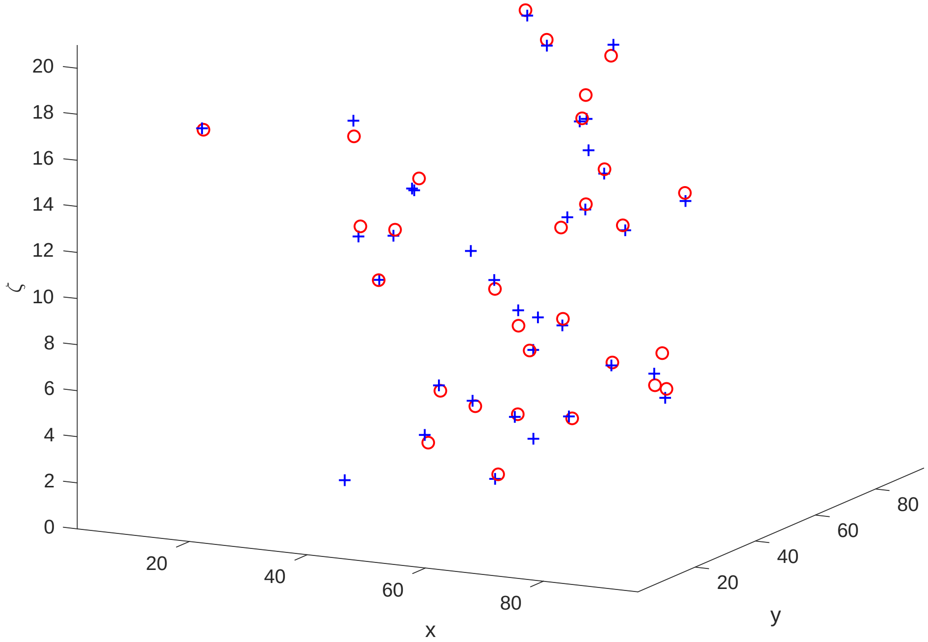
Numerical Result



Observed Image



Estimated location in 2D



"o": ground truth

"+": estimated location

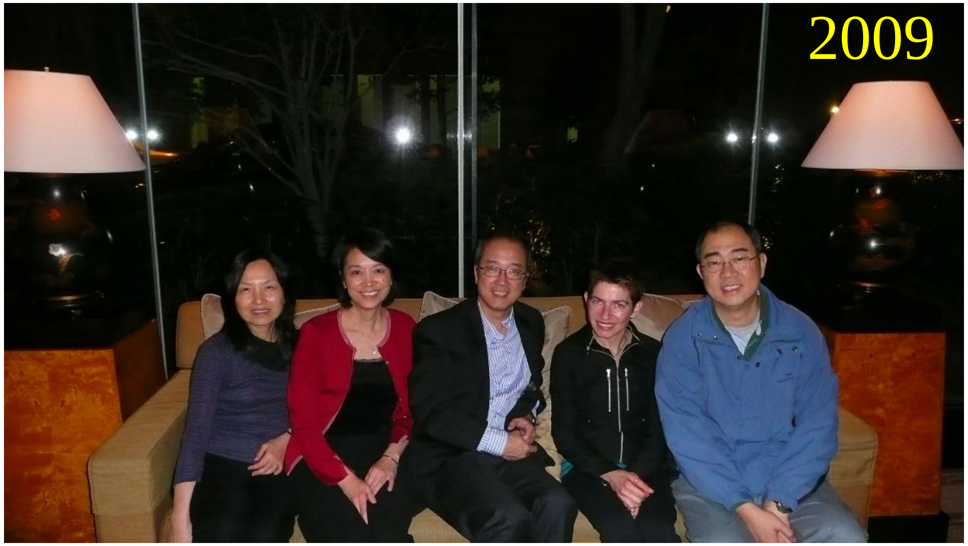
Recall and Precision Accuracies

	ℓ_2-ℓ_1		ℓ_2-NC		KL-ℓ_1		KL-NC	
Points	Recall	Prec.	Recall	Prec.	Recall	Prec.	Recall	Prec.
5	**100.00**	68.91	97.60	89.15	98.93	58.64	**100.00**	**97.52**
10	**99.60**	55.95	94.80	83.51	99.40	65.24	99.40	**93.69**
15	98.67	56.28	92.80	84.77	**98.93**	58.64	98.40	**88.60**
20	97.70	56.50	95.20	80.92	**98.10**	57.82	97.70	**87.49**
30	96.00	55.74	93.93	77.77	94.00	56.22	**96.20**	**79.75**
40	93.80	52.68	**95.40**	59.34	93.70	54.29	95.00	**73.35**

- Recall rate: $\frac{\textbf{Number of identified true positive emitters}}{\textbf{Number of all true emitters}}$
- Precision rate: $\frac{\textbf{Number of identified true positive emitters}}{\textbf{Number of all emitters identified by algorithm}}$

Wang, C., Nikolova, Plemmons, and Prasad, arXiv 1804.04000.

Thank You Mila!



Thank You Mila!

We forever miss you!