Surface tension implementation, verification and validation for separated two-phase flows

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Outline

1. Surface tension implementation in CADYF
2. Verification and validation
   - Water droplets shapes under the effect of gravity
   - Water droplets oscillation
3. Numerical Simulation of Kelvin-Helmholtz Instability
   - Model presentation
   - Results

Outline

About CADYF

- Developed at Polytechnique Montréal.
- Stands for *Computer-Assisted DYnamics of Fluids*
- Finite element program for analysis of planar, axi-symmetric, axi-symmetric swirl and three dimensional Navier-Stokes flows.
- Particularities :
  - Transient simulations with adaptive time step and order.
  - Two-phase flows (fluid-structure and fluid-fluid) using interface tracking method.
  - Monolithic resolution between phases and pseudo-solid mesh deformation.

Surface tension calculation

- Interface curvature is computed using the following relation :

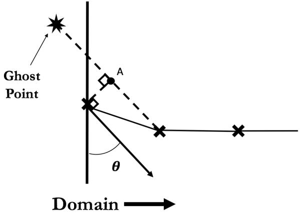
$$\kappa = \frac{x'y'' - y'x''}{(x'^2 + y'^2)^{3/2}}$$

where the derivatives are computed using parametric finite differences.

- τ_σ is applied using reaction method.

$$\tau_\sigma = \sigma \kappa h_e$$

- Imposition of contact angles :



Outline

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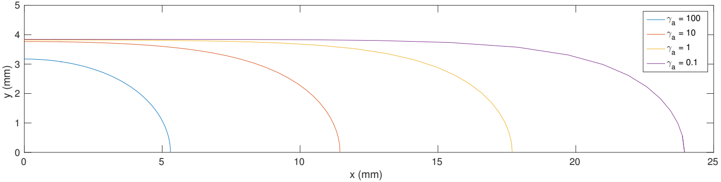
Theoretical Background

- Young-Laplace equation.

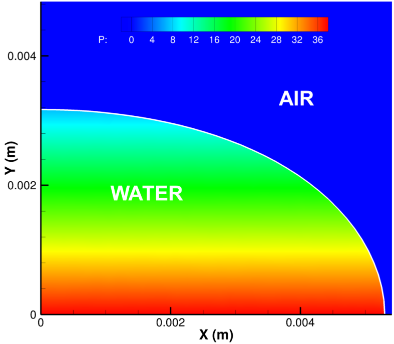
$$\gamma(r, r', r'') = \gamma_a + \beta[r_a - cos(\theta)] \quad with \quad \beta = \frac{\Delta\rho g}{\sigma}$$

- Solved with ODE45 from *Matlab*.



Numerical simulation

- Transient simulation with CADYF using the shape from Y-L equation as initial interface shape.
- Displacement of the interface is less than 5.10^{-3} (mm) for a mesh of 66 elements at the interface.



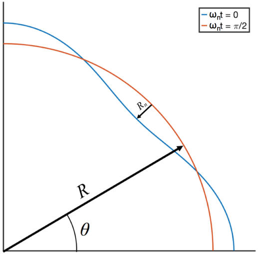
Outline

Theoretical Background

- Shape of an oscillating droplet :

$$r(t,\theta) = \sqrt{R^2 - \frac{R_n(t)^2}{2}} + R_n(t)\cos(n\theta)$$

where R is the steady droplet radius and $R_n(t) = r_n \sin(\omega_n t)$ is the oscillation of the droplet.



Theoretical Background (Fundamental Frequency)

- Potential energy of a 2D oscillating droplet :

$$E_p = \sigma L_n \rightarrow k_{eq} = \frac{\sigma\pi}{R}(n^2-1)$$

- Kinetic energy of a 2D oscillating droplet :

$$E_k = \frac{\rho}{2}\int_0^{2\pi}\left[\Phi\frac{\mathrm{d}\Phi}{\mathrm{d}r}\right]_{r=R} R\mathrm{d}\theta$$

$$\rightarrow m_{eq} = \frac{(\rho_1+\rho_2)\pi R^2}{n}$$

where Φ is the velocity potential.

- Frequency of a 2D oscillating droplet.

$$\omega_n = \frac{k_{eq}}{m_{eq}} = (n^3-n)\frac{\sigma}{(\rho_1+\rho_2)R^3}$$

n	Fundamental Frequency (Hz)
2	20.8
4	65.8
6	123.1
8	190.7

Same relation as Lord Rayleigh (1879).

Theoretical Background (Equivalent Damping)

- From the Batchelor formula we find the dissipation power in the global system :

$$P_d = c_{eq}\dot{R}_n^{\,2} = 2\mu_1 \int_{\Omega_1} e^1_{ij} e^1_{ij} dV + 2\mu_2 \int_{\Omega_2} e^2_{ij} e^2_{ij} dV$$

$$with \quad e^k_{ij} = \frac{1}{2}\left(\frac{\partial u^k_i}{\partial x_j} + \frac{\partial u^k_j}{\partial x_i}\right)$$

which gives us :

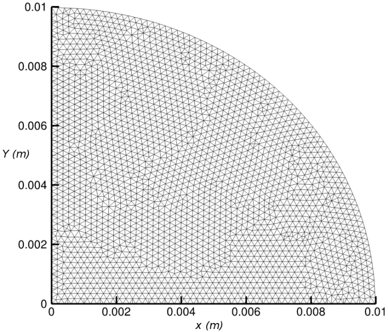
$$\zeta = (2n-2)\sqrt{\frac{n}{n^2-1}}Oh$$

$$avec \quad Oh = \frac{2\pi(\mu_1+\mu_2)}{(\rho_1+\rho_2)R\sigma}$$

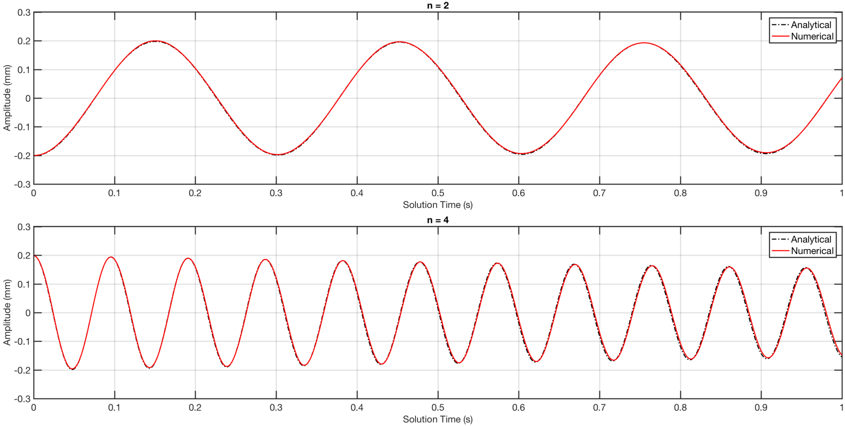
n	Damping Ratio ζ
2	0.192 %
4	0.365 %
6	0.488 %
8	0.588 %

Numerical Simulation

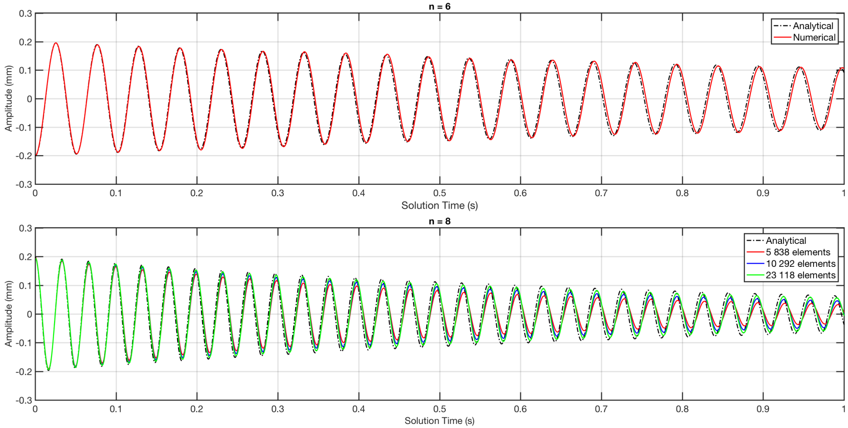
- Transient simulation with CADYF (water/air) using the fully deformed interface as initial shape.
- 23 000 stabilized P1-P1 elements (80 on the interface) was found satisfactory.



Numerical Simulation : Results



Numerical Simulation : Results



Numerical Simulation : Animation

9 times slowed down

Outline

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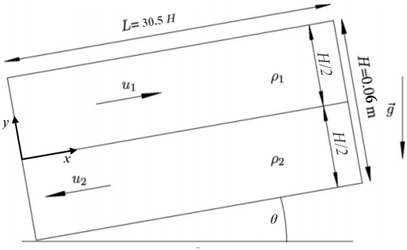
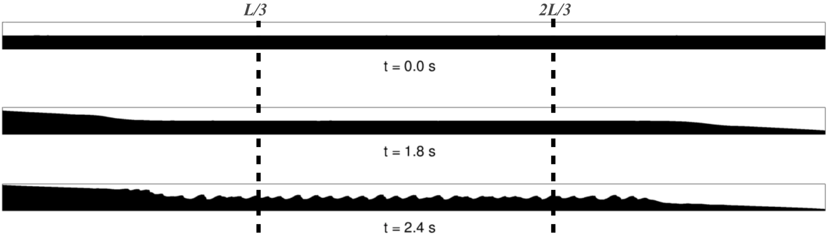
Thorpe's experiment

- Comparison with Thorpe's experiment (1969) :
  $\rho_1 = 780 kg/m^3$
  $\mu_1 = 1.5 mPa/s$
  $\rho_2 = 1000 kg/m^3$
  $\mu_2 = 1.0 mPa/s$
  $\sigma = 0.04 N/m$
  $\theta = 4.1°$





Dimensional analysis

- 5 dimensionless numbers ($U_r = \sqrt{gH}$) :

$$\frac{\rho_1}{\rho_2}; \quad \frac{\mu_1}{\mu_2}; \quad \theta; \quad \frac{\rho_1 U_r^2 H}{\sigma} = We; \quad \frac{\rho_1 U_r H}{\mu_1} = Re$$

- Dimensionless amplitude :

$$\alpha^* = \frac{\alpha}{H} \quad with \quad \alpha = max[y(x_i)] \quad where \quad x_i \in [L/3, 2L/3]$$

- Dimensionless time :

$$t^* = t\sqrt{\frac{g}{H}}$$

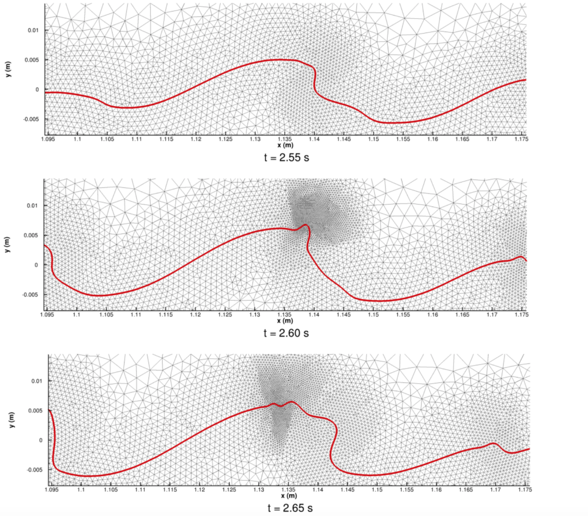
- Dimensionless wavelength :

$$\lambda^* = \frac{\lambda}{H}$$

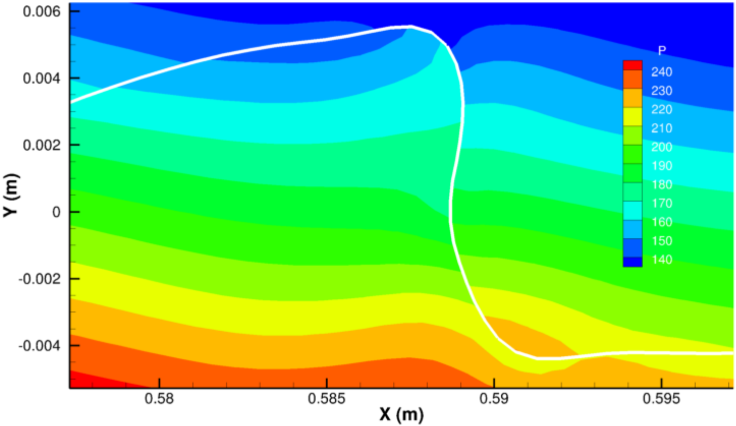
- Thorpe's experiment :

$$We = 689 \quad and \quad Re = 23937$$

Remeshing

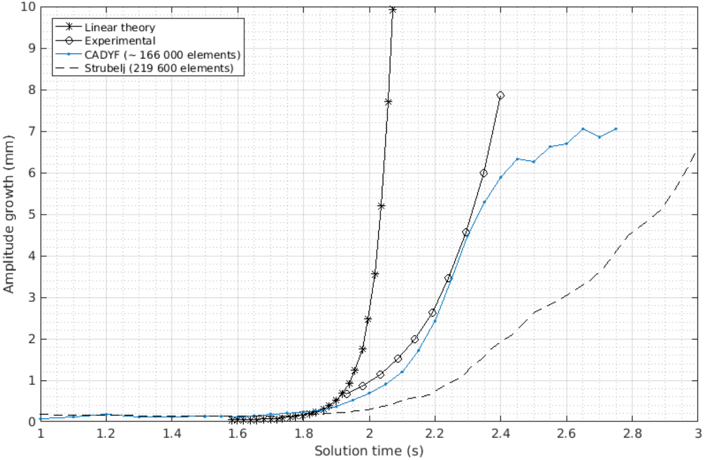


Surface Tension



Outline

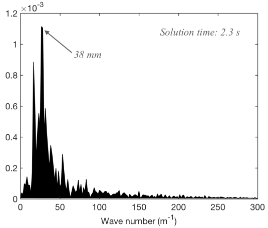
Results comparison



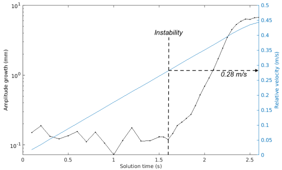
Results comparison

- Most unstable wave length :

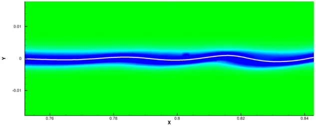
Linear Theory	$\lambda_c = 27mm$
Experimental	$\lambda_c = 25 - 45mm$
Numerical	$\lambda_c \approx 38mm$
Štrubelj	$\lambda_c \approx 40mm$



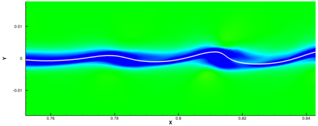
- Instability seems to appear at a relative velocity of $0.28m/s$ which is larger than linear theory prediction ($U_c \approx 0.2m/s$)



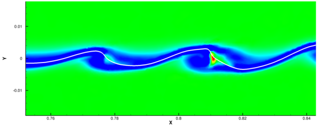
Vorticity field



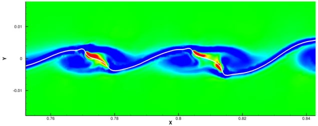
t = 2.1 s



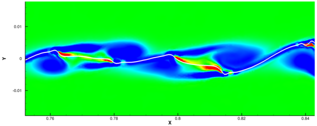
t = 2.2 s



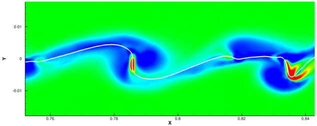
t = 2.3 s



t = 2.4 s

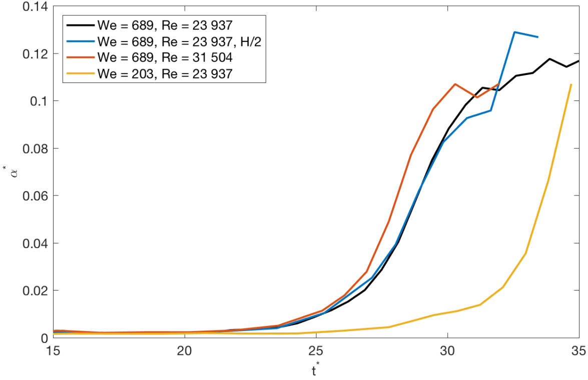


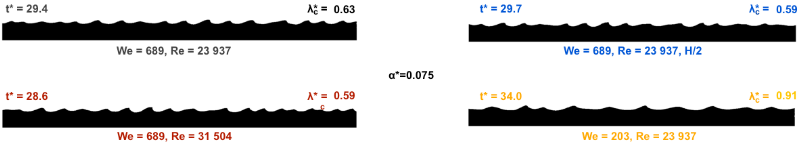
t = 2.5 s



t = 2.6 s

Similarity


We = 689, Re = 23 937
We = 689, Re = 23 937, H/2
We = 689, Re = 31 504
We = 203, Re = 23 937
α*
t*
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
15
20
25
30
35


t* = 29.4
λ*c = 0.63
We = 689, Re = 23 937
t* = 29.7
λ*c = 0.59
We = 689, Re = 23 937, H/2
α*=0.075
t* = 28.6
λ*c = 0.59
We = 689, Re = 31 504
t* = 34.0
λ*c = 0.91
We = 203, Re = 23 937

References

[1] L. Rayleigh. On the capillary phenomena of jets. Proc. R. Soc. London, 29(196-199) :71-97, 1879.
[2] G. K. Batchelor. An introduction to fluid dynamics. Cambridge university press, New York, 2000.
[3] S. A. Thorpe. Experiments on instability of stratified shear flows : immiscible fluids. JFM, 39(1) :25-48, 1969.
[4] L. Štrubelj et I. Tiselj. Numerical simulations of basic interfacial instabilities with incompressible two-fluid model. Nuclear Engineering and Design, 241 :1018-1023, 2011.

Questions ?