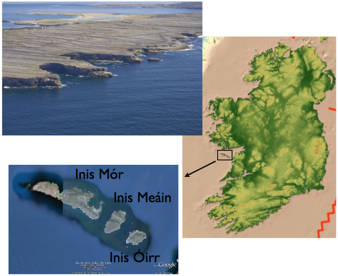
Mechanics of large boulder creation due to wave impacts

James G. Herterich, Rónadh Cox, Frédéric Dias

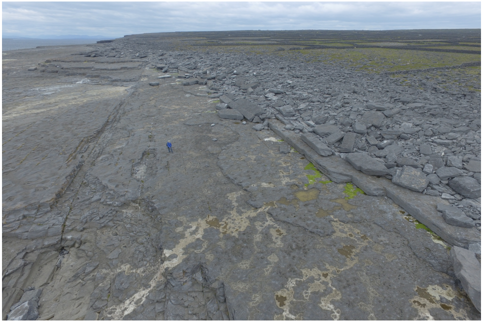
University College Dublin, Williams College, Paris–Saclay

Aran Islands Project



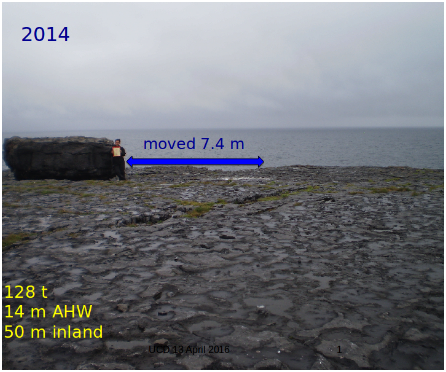
- Three islands exposed to Atlantic storms
- Centuries since last tsunami event

- Cliff-top storm deposits
- Annually observed boulder movement



Boulder Movement

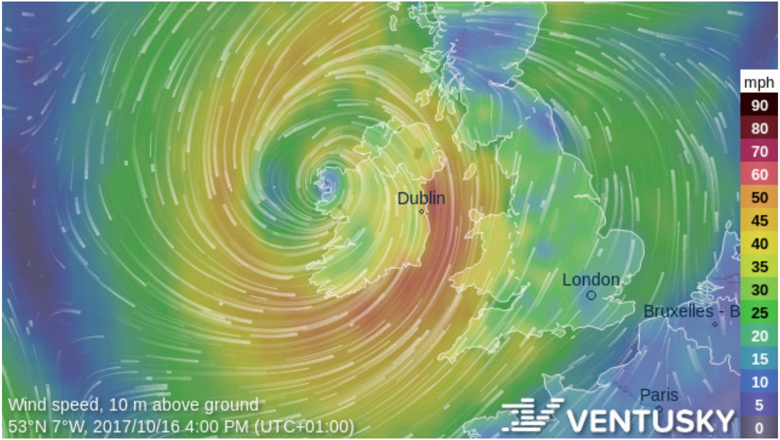
Boulder Movement



Extreme Waves

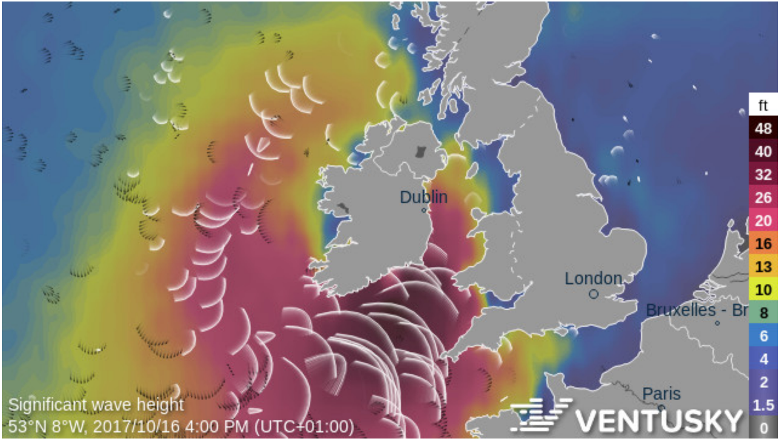
Extreme Waves

Ophelia



Dublin
London
Bruxelles - B
Paris
mph
90
80
70
60
50
45
40
35
30
25
20
15
10
5
0
Wind speed, 10 m above ground
53°N 7°W, 2017/10/16 4:00 PM (UTC+01:00)
VENTUSKY

Ophelia



Dublin
London
Bruxelles - Br
Paris
Significant wave height
53°N 8°W, 2017/10/16 4:00 PM (UTC+01:00)
VENTUSKY
ft
48
40
32
26
20
16
13
10
8
6
4
2
1.5
0

Ophelia

Boulder movement

...?

To be investigated!

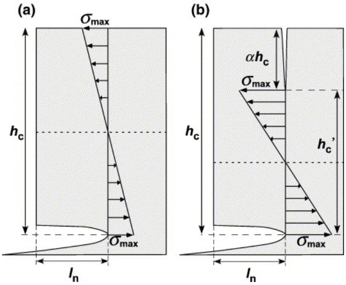
Question

What happens when waves interact with cliffs?

Cliff Collapse

Gravity Loads

- Notch eroded
- Gravity load stress
- Crack develops
- Load increases
- Cliff collapse

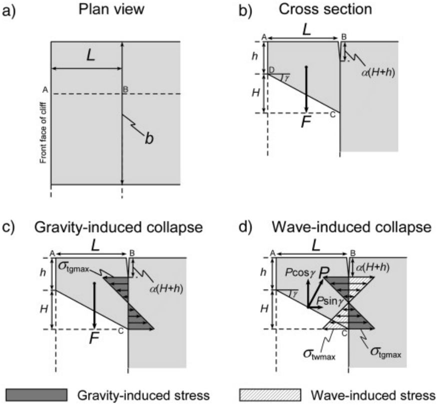


Kogure *et al.*, 2006

Cliff Collapse

Wave Loads

- Gravity & wave load stress
- Stresses pull crack apart
- Cliff collapse



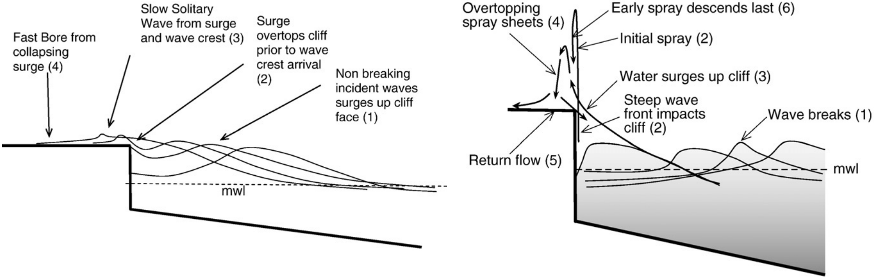
Kogure *et al.*, 2012

Block Erosion

Wave Impacts & High Pressure Loads → Hydraulic Fracture

- Overtopping waves
- Non-breaking wave impact

- Runup jets
- Breaking wave impact



(Hansom *et al.*, 2008)

Block Erosion Ex 1

dimensions: 5.7 m × 2.0 m × 0.8 m
mass: 22.3 t

16 m above high water
166 m inland

Block Erosion Ex 2

Block Erosion Ex 3

Literature References

Hall *et al.*, 2006

blocks are quarried from the cliff surface

Hansom *et al.*, 2008

the waves are capable of overtopping 10–30 m high cliffs and generate cliff-top forces sufficient to fracture bedrock and to detach and lift boulders as large as 277 m^3

Literature References

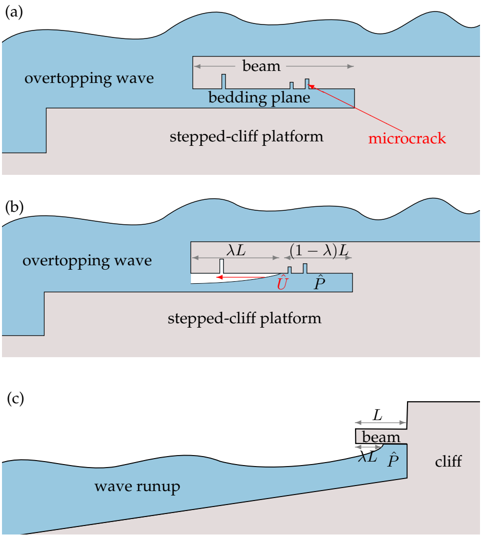
Fichaut & Suanez, 2011

[overtopping] is also capable of quarrying blocks from the cliff face close to the edge and from rock steps on the cliff top, promoting further rock fracturing

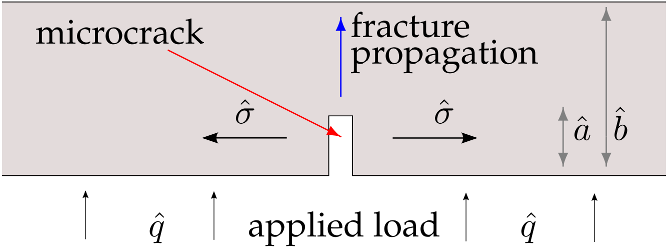
Erdmann *et al.*, 2017

Stratified limestone bedrock with bedding planes and joint patterns allows strong wave fracturing into platy boulders, deposited in ridges .... Those broken from bedrock are platy from limestone strata with constant thickness

Setup: Exposed Beams

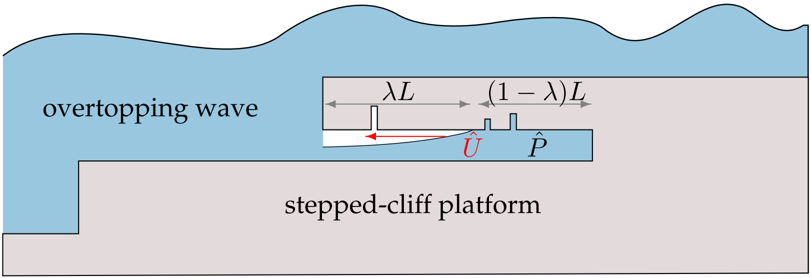


Fracture



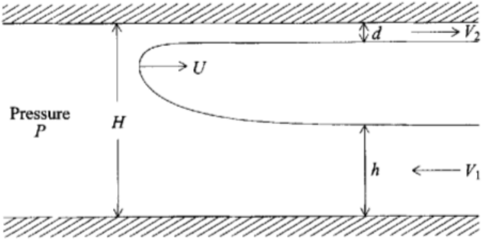
- Normal load from fluid
- Load induces bending stress
- Stress amplified in cracks and weaknesses
- Cracks propagate to complete fracture

Filling Flows



- Dynamics of crack filling determines load
- Fluid rushes in and fills from inside out

Filling Flows



Peregrine & Kalliadasis, 1996

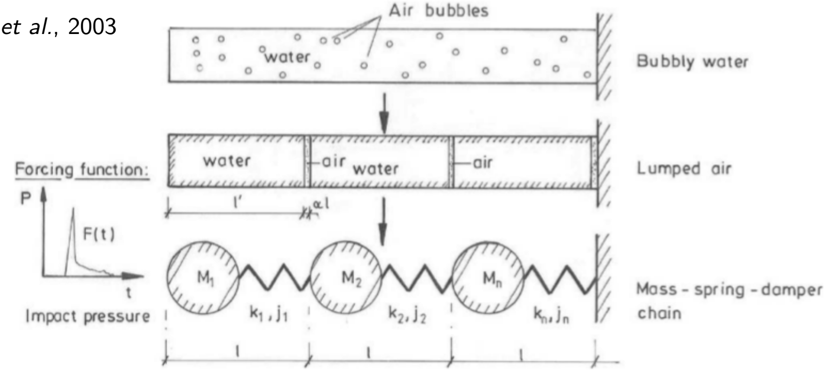
$$P = \frac{V_1^2 k^2}{1 - k^2} \qquad k = \sqrt{h/H}$$

$$U = \frac{V_1 k^2}{1 - k^2}$$

Filling Flows

Full air/water mixtures propagate pressure pulses when further impacts occur

Müller *et al.*, 2003

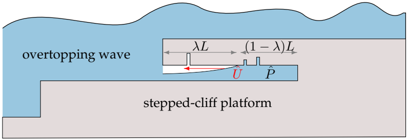


Solid Response

Assumptions

- Piecewise spatially constant load – fraction λ unfilled
- Neglect gravity load
- Quasi-static
- Euler–Bernoulli & Timoshenko beam theories
- Griffith Theory for Mode I brittle fracture

$$q = \begin{cases} 0 & 0 < x < \lambda \\ P(1-\lambda) & \lambda \leq x < 1 \end{cases}$$



Deformation

Euler–Bernoulli

$$(u_x, u_y, u_z) = (0, 0, w(x))$$

$$\frac{\partial^4 w}{\partial x^4} = \alpha^2 q$$

$$w(1) = 0, \qquad \frac{\partial w}{\partial x}(1) = 0,$$
$$\frac{\partial^2 w}{\partial x^2}(0) = 0, \qquad \frac{\partial^3 w}{\partial x^3}(0) = 0.$$

$$M = -\frac{\partial^2 w}{\partial x^2} \qquad \sigma = \zeta M z$$

Timoshenko

$$(u_x, u_y, u_z) = (-z\phi(x), 0, w(x))$$

$$\frac{\partial^3 \phi}{\partial x^3} = \alpha^2 q,$$
$$\frac{\partial w}{\partial x} = \phi - \omega^2 \frac{\partial^2 \phi}{\partial x^2}$$

$$w(1) = 0, \qquad \phi(1) = 0,$$
$$\frac{\partial \phi}{\partial x}(0) = 0, \quad -\phi(0) + \frac{\partial w}{\partial x}(0) = 0.$$

$$M = -\frac{\partial \phi}{\partial x} \qquad \sigma = \zeta M z$$

$$\alpha^2 = \hat{P} L^4 / \delta E I \qquad \omega^2 = E I / \kappa A G L^2 \qquad \zeta = \delta^2 E / \hat{P}$$

Deformation

Euler–Bernoulli

$$w = \begin{cases} \frac{\alpha^2 q}{24}(1-\lambda)^3(3-4x+\lambda) & 0 < x < \lambda \\ \frac{\alpha^2 q}{24}\left[(x-\lambda)^4 + (1-\lambda)^3(3-4x+\lambda)\right] & \lambda \leq x < 1 \end{cases}$$

Timoshenko

$$\phi = \begin{cases} \frac{\alpha^2 q}{6}(\lambda-1)^3 & 0 < x < \lambda \\ \frac{\alpha^2 q}{6}(x-1)\left[1+x+x^2-3\lambda-3x\lambda+3\lambda^2\right] & \lambda \leq x < 1 \end{cases}$$

$$w = \begin{cases} \frac{\alpha^2 q}{24}(1-\lambda)^2\left[3-4x(1-\lambda)-2\lambda-\lambda^2+12\omega^2\right] & 0 < x < \lambda \\ \frac{\alpha^2 q}{24}(1-x)\left[3-x^3-x^2(1-4\lambda)-8\lambda+6\lambda^2+12\omega^2 \right. \\ \qquad \left. -24\lambda\omega^2 - x(1-4\lambda+6\lambda^2-12\omega^2)\right] & \lambda \leq x < 1 \end{cases}$$

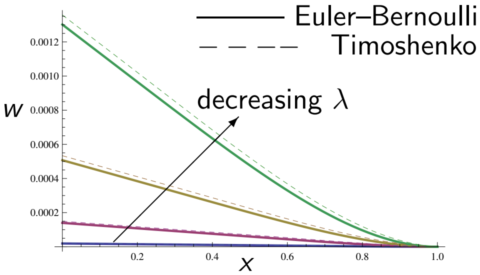
Stress

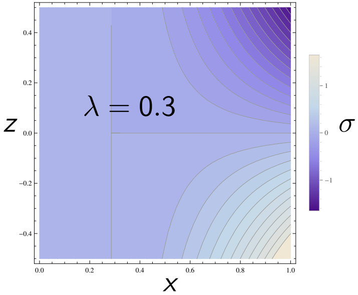
Euler–Bernoulli & Timoshenko

$$\sigma = \begin{cases} 0 & 0 < x < \lambda \\ -\beta^2 P \frac{(1-\lambda)(x-\lambda)^2 z}{2} & \lambda \leq x < 1 \end{cases}$$

$$\boxed{\beta^2 = \delta L^4 / Ib \sim \delta^{-2}}$$

Stress





$p = 0.1 = 1/10 \times$ Fracture stress $\rightarrow \sigma > 1$

Fracture

Assumptions

- Rock is brittle
- Tensile stress of a Mode I crack

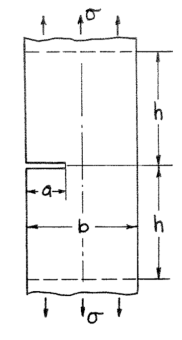
Griffith Theory

$$\sigma_f = \sqrt{\frac{2E\gamma}{\pi a}} \rightarrow \sqrt{\frac{1-\nu^2}{\pi a}} K_{Ic}$$

$$\sigma_c = \frac{K_I}{\sqrt{2\pi r}} f(\theta)$$

$$K_I = \sigma\sqrt{\pi a} \sum_{j=0}^{4} S_j (a/b)^j$$

$\{S\} = \{$ 1.122, -0.231, 10.55, -21.71, 30.382 $\}$



(Tada *et al.*, 1973)

Fracture

Assumptions

- Rock is brittle
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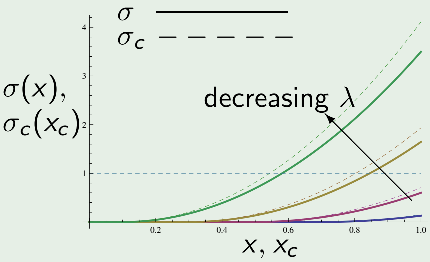
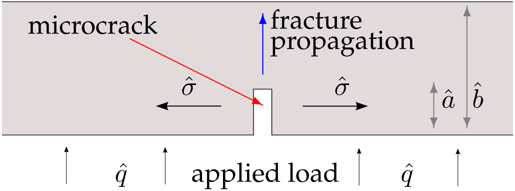


Pressure for Fracture

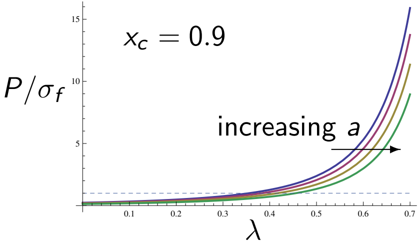


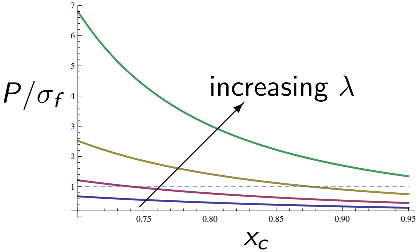
$$\sigma = \begin{cases} 0 & 0 < x < \lambda \\ \beta^2 q \frac{(x-\lambda)^2}{4} & \lambda \leq x < 1 \end{cases}$$

$$\beta^2 \sim \delta^{-2}$$

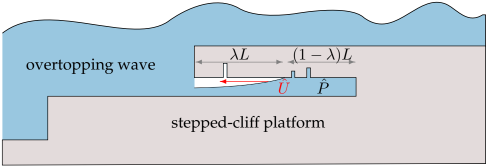
$$P > \sigma_f \left[\frac{(1-\lambda)(x_c-\lambda)^2}{4\sqrt{2}\,\delta^2} \sum_{j=0}^{4} S_j \left(\frac{a}{b}\right)^j \right]^{-1} \qquad \lambda < x_c < 1$$

Pressure for Fracture

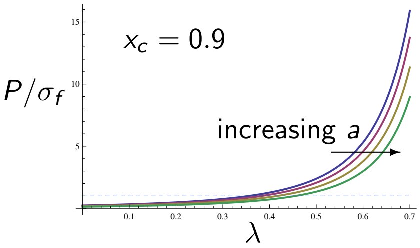


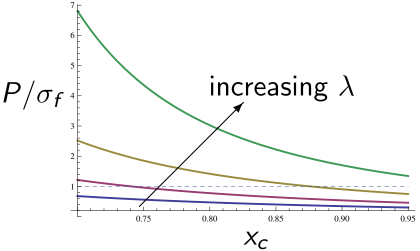


$$\frac{P}{\sigma_f} = \left[\frac{(1-\lambda)(x_c-\lambda)^2}{4\sqrt{2}\,\delta^2} \sum_{j=0}^{4} S_j \left(\frac{a}{b}\right)^j \right]^{-1} \qquad \lambda < x_c < 1$$

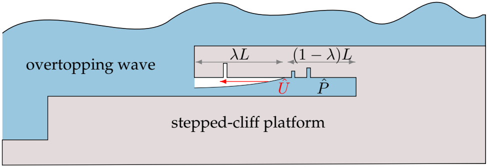


Pressure for Fracture





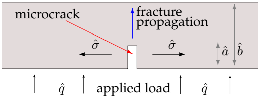
$$\frac{P}{\sigma_f} = \left[\frac{(1-\lambda)(x_c-\lambda)^2}{4\sqrt{2}\,\delta^2} \sum_{j=0}^{4} S_j \left(\frac{a}{b}\right)^j \right]^{-1} \qquad \lambda < x_c < 1$$



Pressure for fracture decreases with

- $(\text{crack position})^{-2}$
- $(\text{loaded area})^{-3}$

Conclusions



- Simple stressed-beam model for boulder creation via hydraulic fracture
- Pressure for fracture can be significantly below fracture stress
- Boulder creation is a unique way of measuring storm and wave power