A cartesian scheme for compressible multimaterials with plasticity

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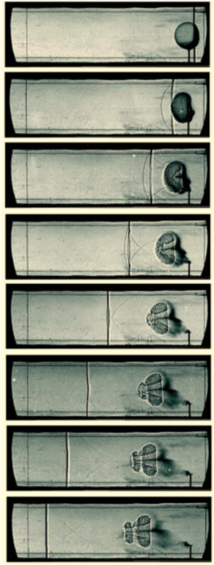
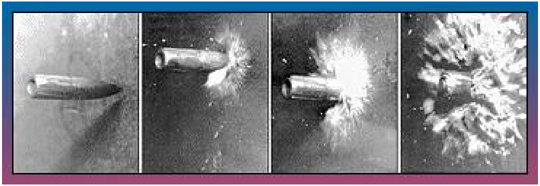
Physical context





Physical context

Objectives

⟶ Modeling the interaction of compressible multimaterials
⟶ Transitory regime
⟶ Plasticity modeling

Difficulties

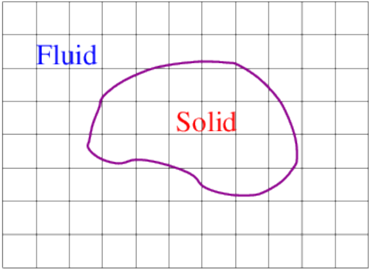
- Different constitutive laws and large density ratios (gas/solid)
- Different formulations (Eulerian/Lagrangian)

⟶ Very challenging problems with many applications

- Shock Bubble interaction
- Impacts, Rebounds ...

IBM and cartesian mesh for FSI

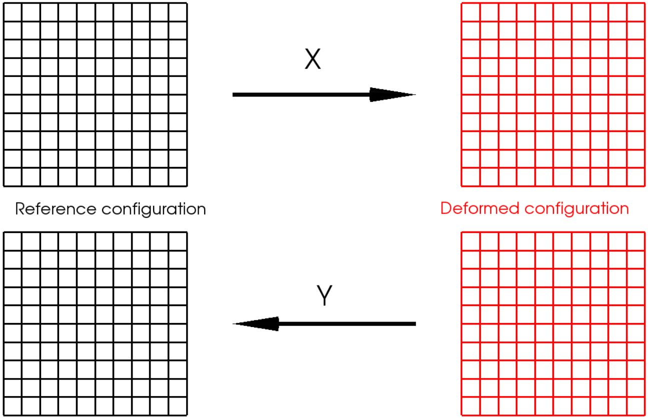
Our approach : Treat the elasticity in an Eulerian way



Advantages of the Eulerian framework

- Unified formulation for the fluid and the solid
- Discretization on a fixed cartesian mesh (3D, parallelization)
- Natural treatment of large deformations

Eulerian elasticity Cottet-Maitre-M 2008



$Y(x, t)$: Backward characteristics. $\qquad Y(X(\xi, t), t) = \xi$

$$\partial_t Y + u \cdot \nabla Y = 0 \qquad [\nabla_\xi X] = [\nabla_x Y]^{-1}$$

Eulerian governing equation

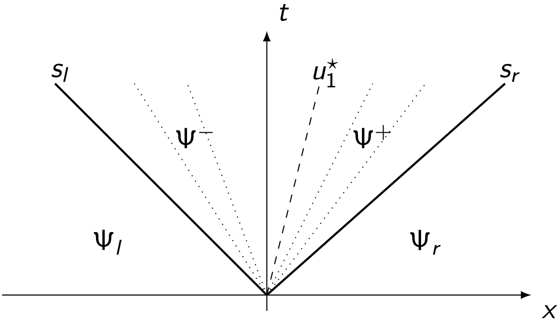
$$\left\{\begin{array}{rcl} \partial_t \rho + \mathrm{div}(\rho u) & = & 0 \\ \partial_t(\rho u) + \mathrm{div}(\rho u \otimes u - \sigma) & = & 0 \\ \partial_t(\nabla Y) + \nabla(u \cdot \nabla Y) & = & 0 \\ \partial_t(\rho e) + \mathrm{div}(\rho e u - \sigma^T u) & = & 0 \end{array}\right.$$

Internal energy (constitutive law to close the system)

$$\varepsilon = \underbrace{\overbrace{\overbrace{\frac{\kappa(s)\rho^{\gamma-1}}{\gamma - 1}}^{\text{Perfect gas}} + \frac{p_\infty}{\rho}}^{\text{Stiffened gas}} + \frac{\chi}{\rho_0}(\mathrm{Tr}(\overline{B}) - 3)}_{\text{NeoHookean elastic solid}}$$

New result : the neohookean model is hyperbolic in 3D

HLLC Riemann Solver for $\mathcal{F}(\Psi_r; \Psi_l)$

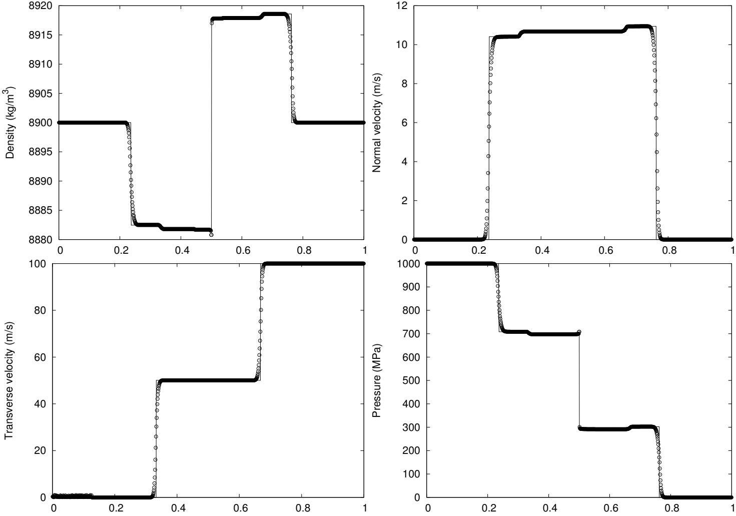


- Two fastest waves s_l and s_r
- Continuity of normal velocity and normal stress
- Rankine-Hugoniot $\longrightarrow \Psi^+$ and Ψ^-
- Sharp treatment at the interface (no oscillations)

Extension to order 2 accuracy in space with a MUSCL scheme

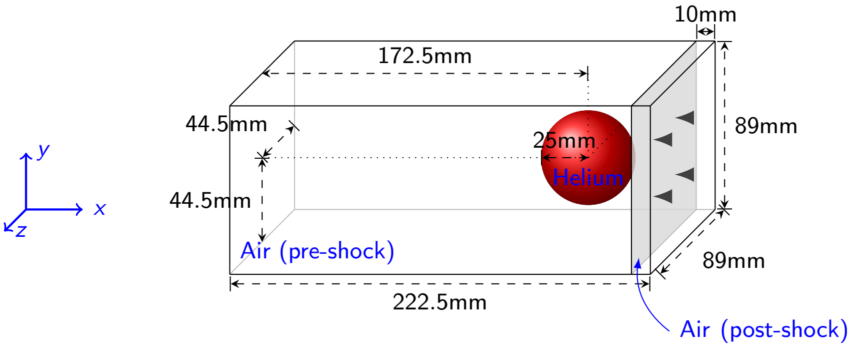
Copper shock tube with shear $N = 1000$

side	ρ	u_1	u_2	p	γ	p_∞	χ	t_{end}
left	8900	0	0	10^9	4.22	$3.42 \cdot 10^{10}$	$5 \cdot 10^{10}$	$5 \cdot 10^{-5}$
right	8900	0	100	10^5	4.22	$3.42 \cdot 10^{10}$	$5 \cdot 10^{10}$	

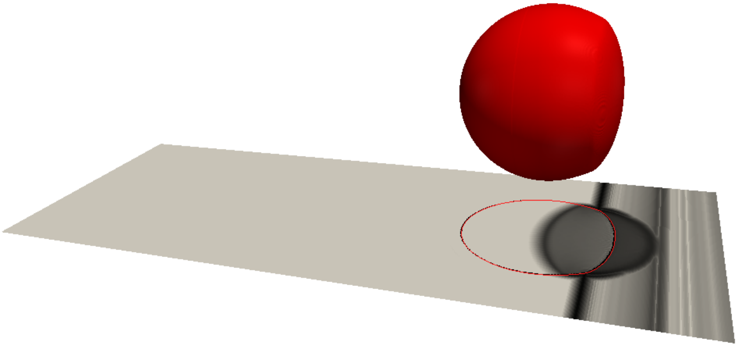


3D Air/Helium shock bubble interaction, $M_a = 1.22$ Mesh=$1000 \times 400 \times 400$

Media	ρ	u_1	p	γ	p_∞	χ
Air (pre-shock)	1.225	0	101325	1.4	0	0
Air (post-shock)	1.6861	-113.534	159059			
Helium	0.2228	0	101325	1.648	0	0



3D Air/Helium shock bubble interaction
Mesh=$1000 \times 400 \times 400$



3D Air/Water shock bubble interaction, $M_a = 1.42$ Mesh=$400 \times 400 \times 480$

<table>
<tr><th>Media</th><th>ρ</th><th>u_1</th><th>p</th><th>γ</th><th>a</th><th>b</th><th>p_∞</th><th>χ</th></tr>
<tr><td>Water (pre-shock)</td><td>1000</td><td>0</td><td>10^5</td><td rowspan="2">4.4</td><td rowspan="2">0</td><td rowspan="2">0</td><td rowspan="2">$6 \cdot 10^8$</td><td rowspan="2">0</td></tr>
<tr><td>Water (post-shock)</td><td>1230</td><td>-432.69</td><td>10^9</td></tr>
<tr><td>Air</td><td>1.2</td><td>0</td><td>10^5</td><td>1.4</td><td>5</td><td>10^{-3}</td><td>0</td><td>0</td></tr>
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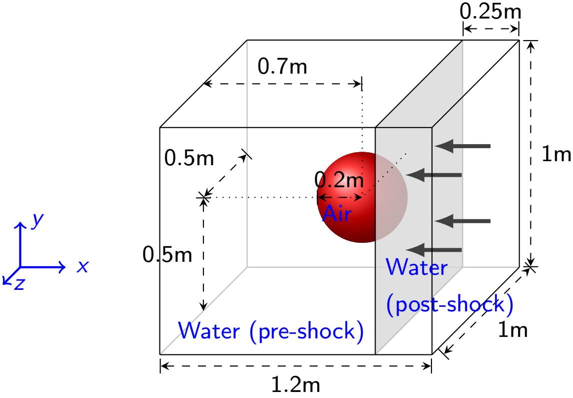
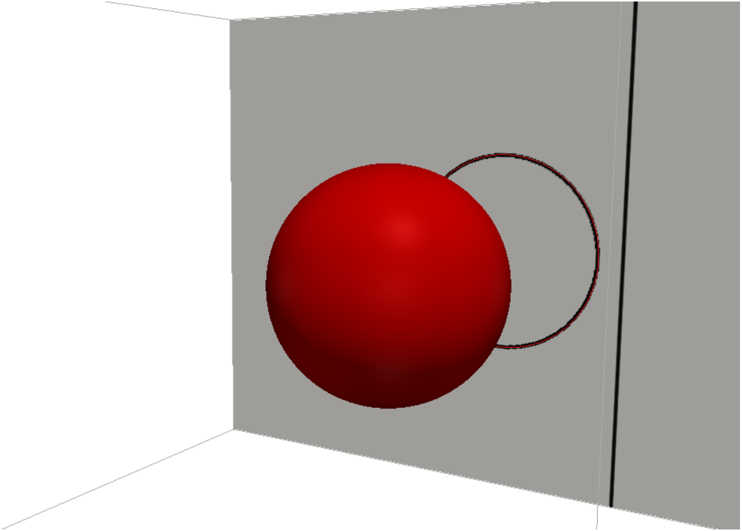


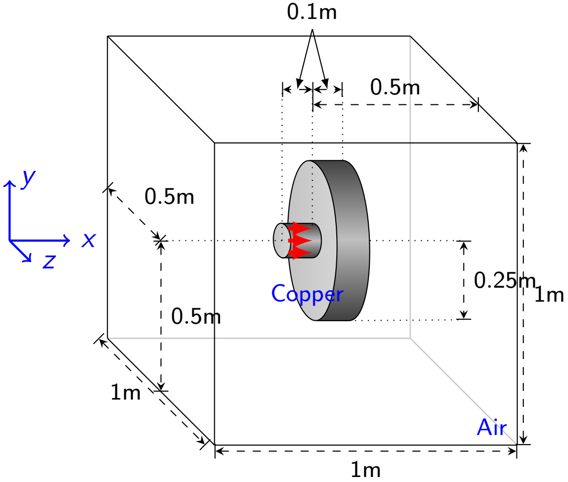
3D Air/Water shock bubble interaction

Mesh=$400 \times 400 \times 480$

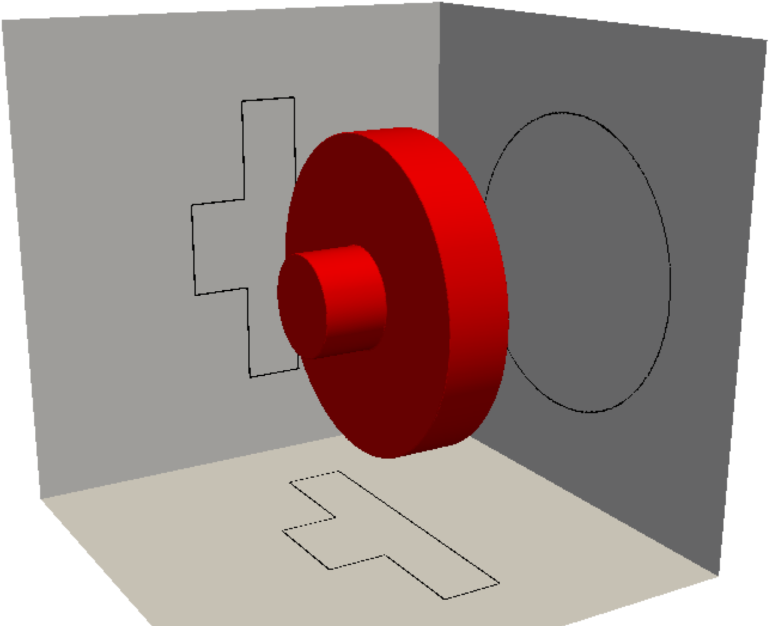


3D Air/Copper impact. Mesh=600^3

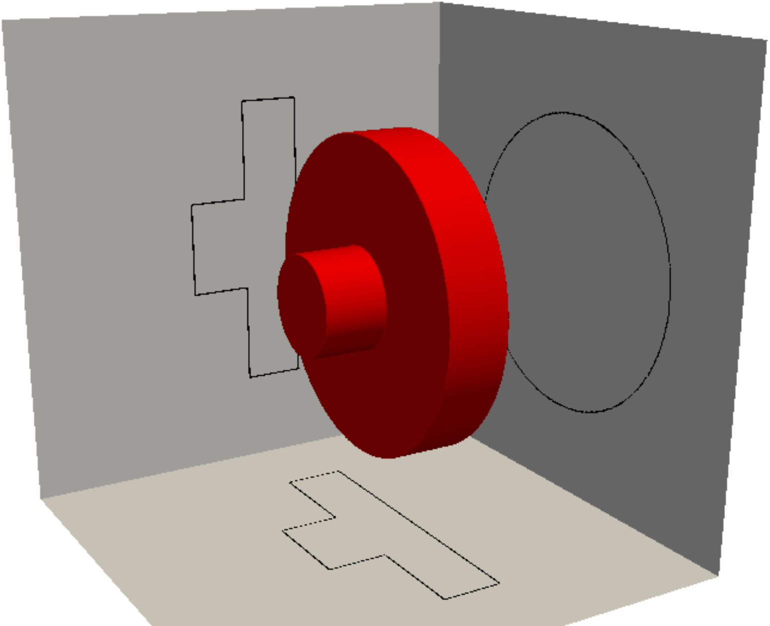
Media	ρ	u_1	p	γ	p_∞	χ
Copper (plate)	8900	0	10^5	4.22	$3.42 \cdot 10^{10}$	$5 \cdot 10^{10}$
Copper (projectile)	8900	800	10^5			
Air	1	0	10^5	1.4	0	0



3D Air/Copper impact Mesh=600^3

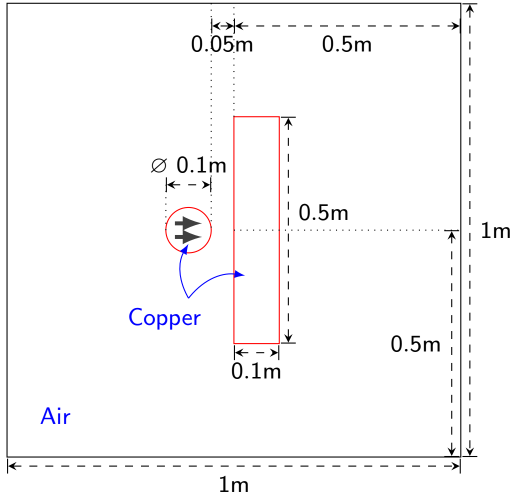


3D Air/Water impact Mesh=600^3

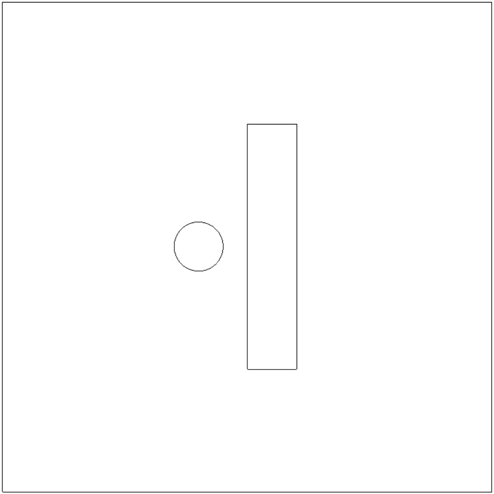


2D Air/Copper rebound. Mesh=4000^2

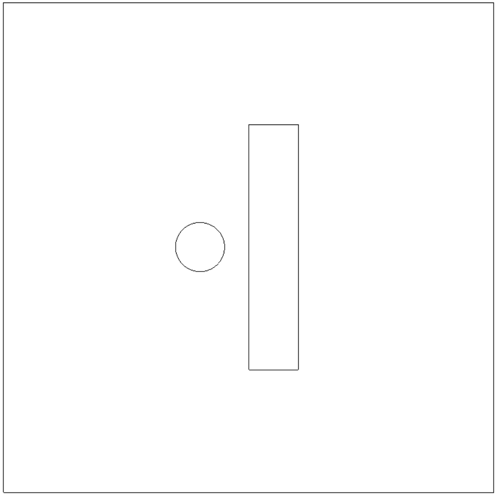
Media	ρ	u_1	p	γ	p_∞	χ
Copper (plate)	8900	0	10^5	4.22	$3.42 \cdot 10^{10}$	$5 \cdot 10^{10}$
Copper (projectile)	8900	800	10^5			
Air	1	0	10^5	1.4	0	0



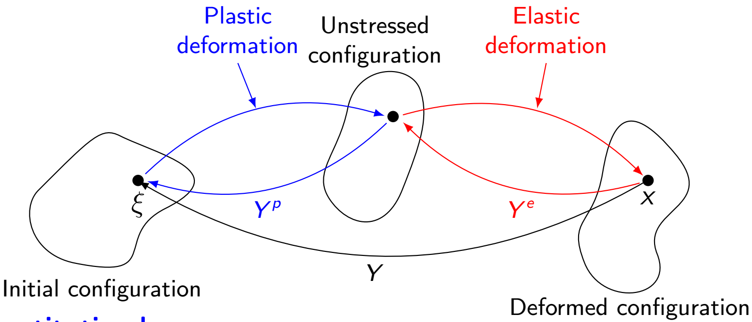
2D Rebound Mesh=4000^2



2D Rebound Mesh=4000^2 bis



Plasticity modeling



Constitutive law

$$\partial_t(\nabla Y^e) + \nabla(u \cdot \nabla Y^e) = \frac{1}{\chi\tau}[\nabla Y^e]\mathrm{dev}(\sigma)$$

with Von Misses criteria $f_{VM}(\sigma) = |\mathrm{dev}(\sigma)|^2 - \frac{2}{3}(\sigma_y)^2$

$$f_{VM}(\sigma) < 0 \longrightarrow \text{elastic regime} \longrightarrow \frac{1}{\tau} = 0$$

$$f_{VM}(\sigma) \geq 0 \longrightarrow \text{plastic regime} \longrightarrow \frac{1}{\tau} \neq 0$$

Properties of the plastic model

Plastic ODE

$$\partial_t(\nabla Y^e) = \frac{1}{\chi\tau}[\nabla Y^e]\mathrm{dev}(\sigma)$$

During the plastic process

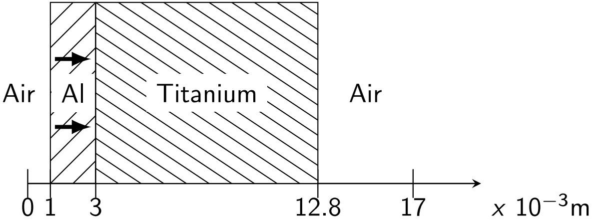
1. the volume is constant
2. the entropy is increasing
3. $\mathrm{dev}(\sigma)$ is decreasing until reaching the yield surface

1 and 2 are classical results of the literature

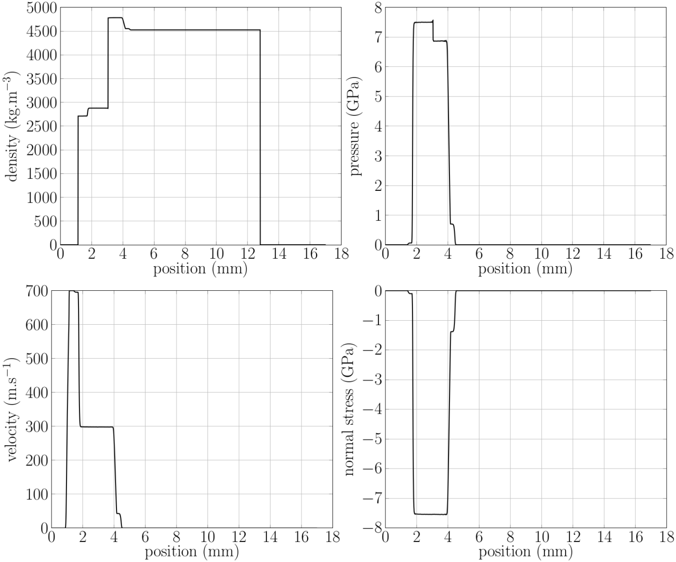
New result : 3 is satified for the NeoHookean model
Numerical method : splitting in time : hyperbolic step + Plastic EDO

1D impact at 700$m.s^{-1}$ mesh=2000

Media	ρ ($kg.m^{-3}$)	γ	p_∞ (GPa)	χ (GPa)	σ_y (GPa)	τ_0 (s)
Air	1	1.4	0	0	–	–
Aluminium (Al)	2712	3.5	32	26	0.06	10^{-9}
Titanium	4527	2.6	44	42	1.03	10^{-8}

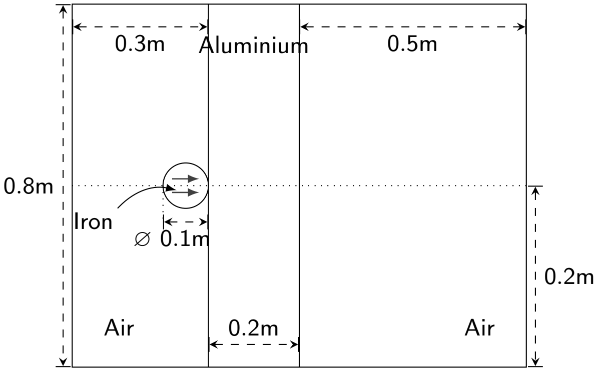


1D impact at $700 m.s^{-1}$ mesh=2000

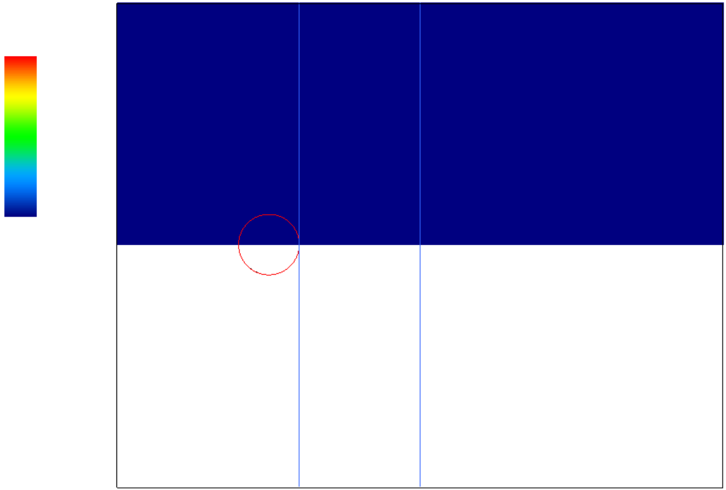


2D impact at $1 km.s^{-1}$ mesh=2000×1600

Media	ρ (kg.m^{-3})	γ	p_∞ (GPa)	χ (GPa)	σ_y (GPa)	τ_0 (s)
Air	1	1.4	0	0	–	–
Iron	7860	3.9	43.6	82	0.2	2.10^{-6}
Aluminium	2712	3.5	32	26	0.06	3.10^{-7}

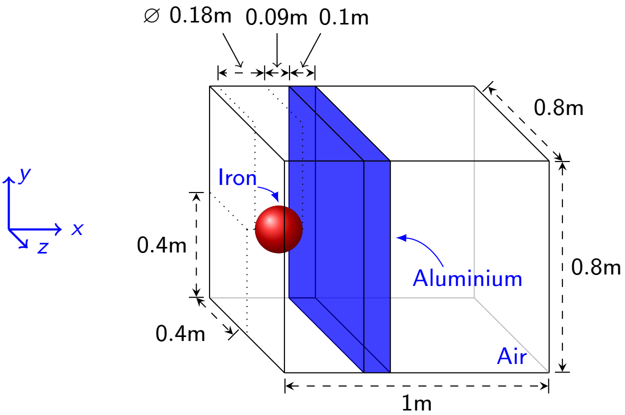


2D impact of Iron on Aluminium plate at $1 km.s^{-1}$ mesh=2000 $\times$ 1600

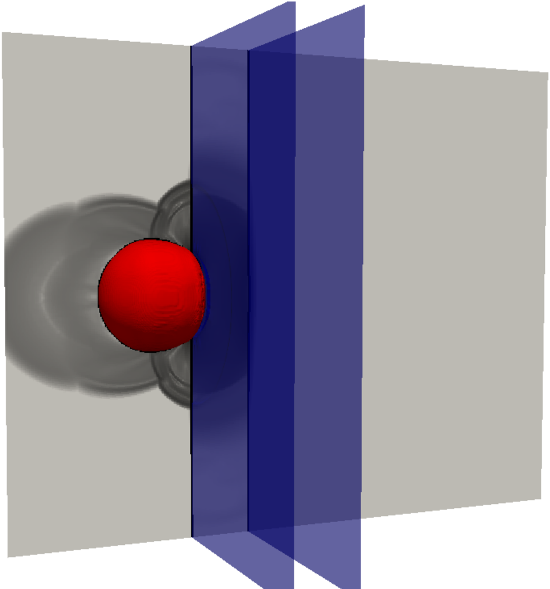


3D impact at $1 km.s^{-1}$ mesh=500×400^2

Media	ρ (kg.m^{-3})	γ	p_∞ (GPa)	χ (GPa)	σ_y (GPa)	τ_0 (s)
Air	1	1.4	0	0	–	–
Iron	7860	3.9	43.6	82	0.2	2.10^{-4}
Aluminium	2712	3.5	32	26	0.06	1.10^{-6}



3D impact of Iron on Aluminium plate at $1 km.s^{-1}$ mesh=500×400^2



Conclusions and perspectives

Conclusions

- New theoretical results on hyperbolocity and plasticity for the neohookean model
- **Simple and robust scheme-> shock bubble interaction, impacts with plasticity in 3D**

Perspectives

- Semi-implicit schemes for low Mach flows
- **Wide range of possible applications**

References

- A simple Cartesian scheme for compressible multimaterials, JCP vol 272 p772-798 (2014)
- A Cartesian scheme for compressible multimaterial models in 3D, JCP vol 313 p121-143 (2016)
- A Cartesian scheme for compressible multimaterial hyperelastic models with plasticity, CiCP vol 22 (5) p1362-1384 (2017)