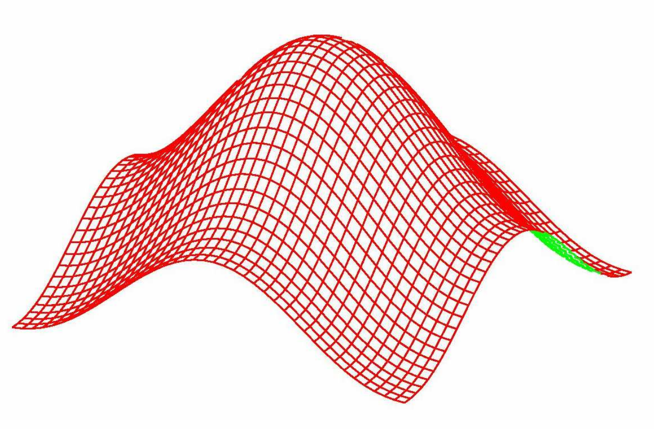
On natural modes in moonpools and gaps

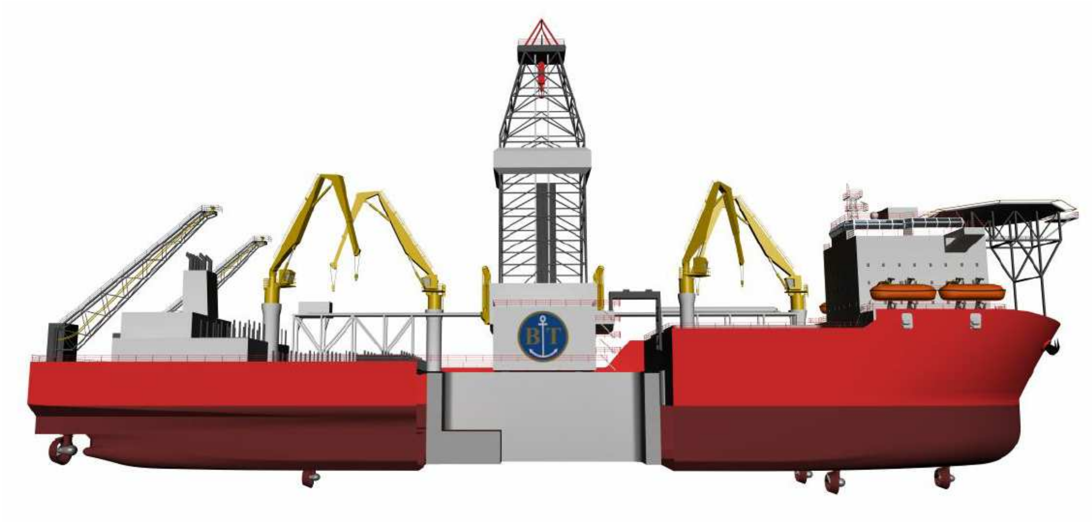
Bernard Molin



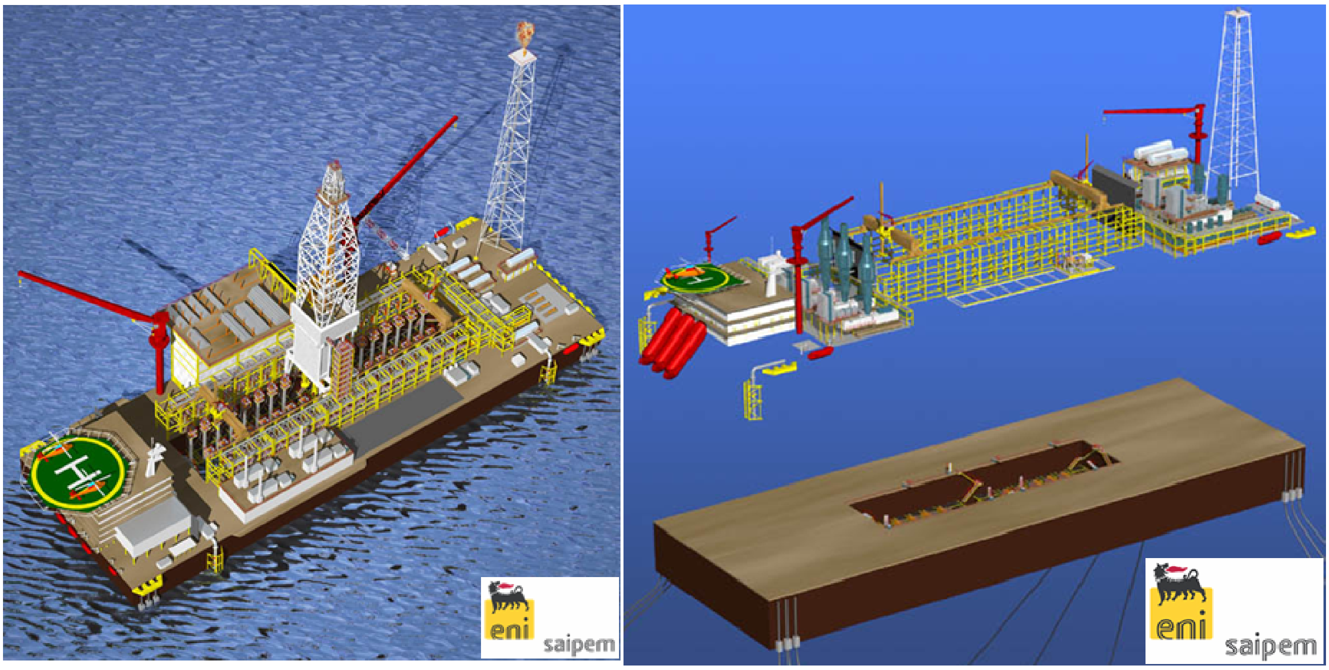
Multiphase 2017
ENS-Cachan, 16 October 2017

- Moonpool = vertical opening through the hull of some ships and offshore structures
  (= tank without bottom)

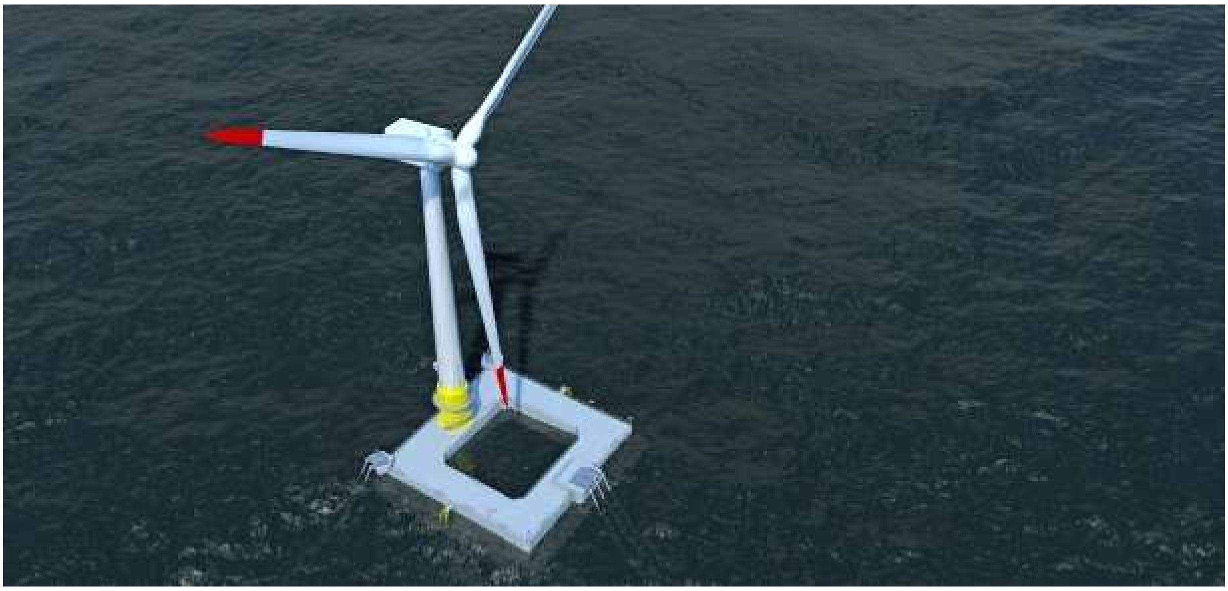
- Gap = space in-between two ships or between ship and quay
  (= narrow moonpool without ends)



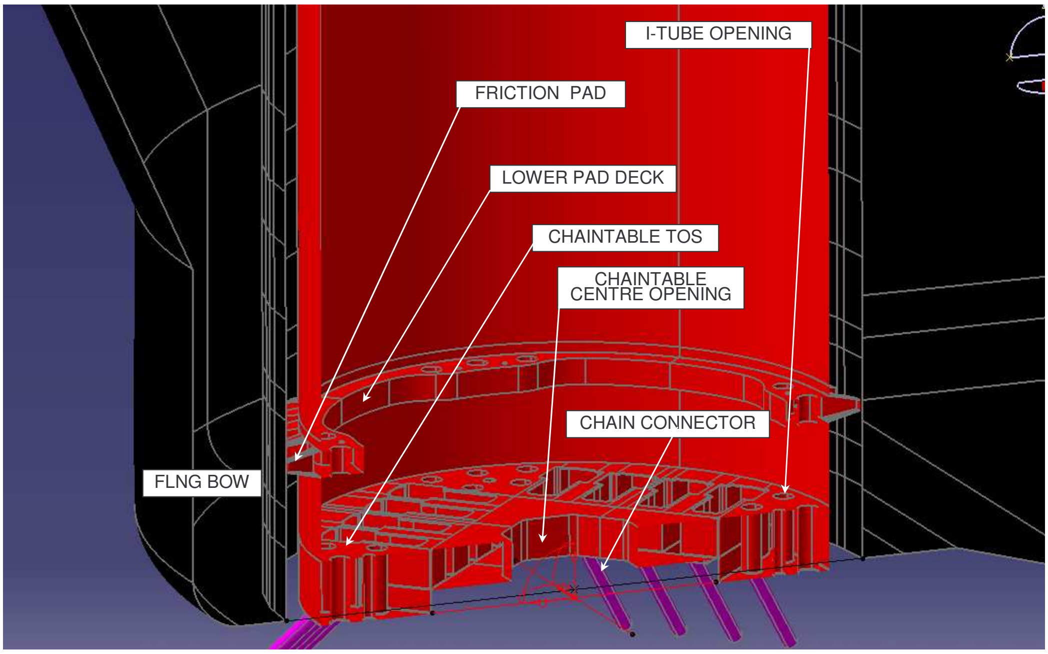
Drillships



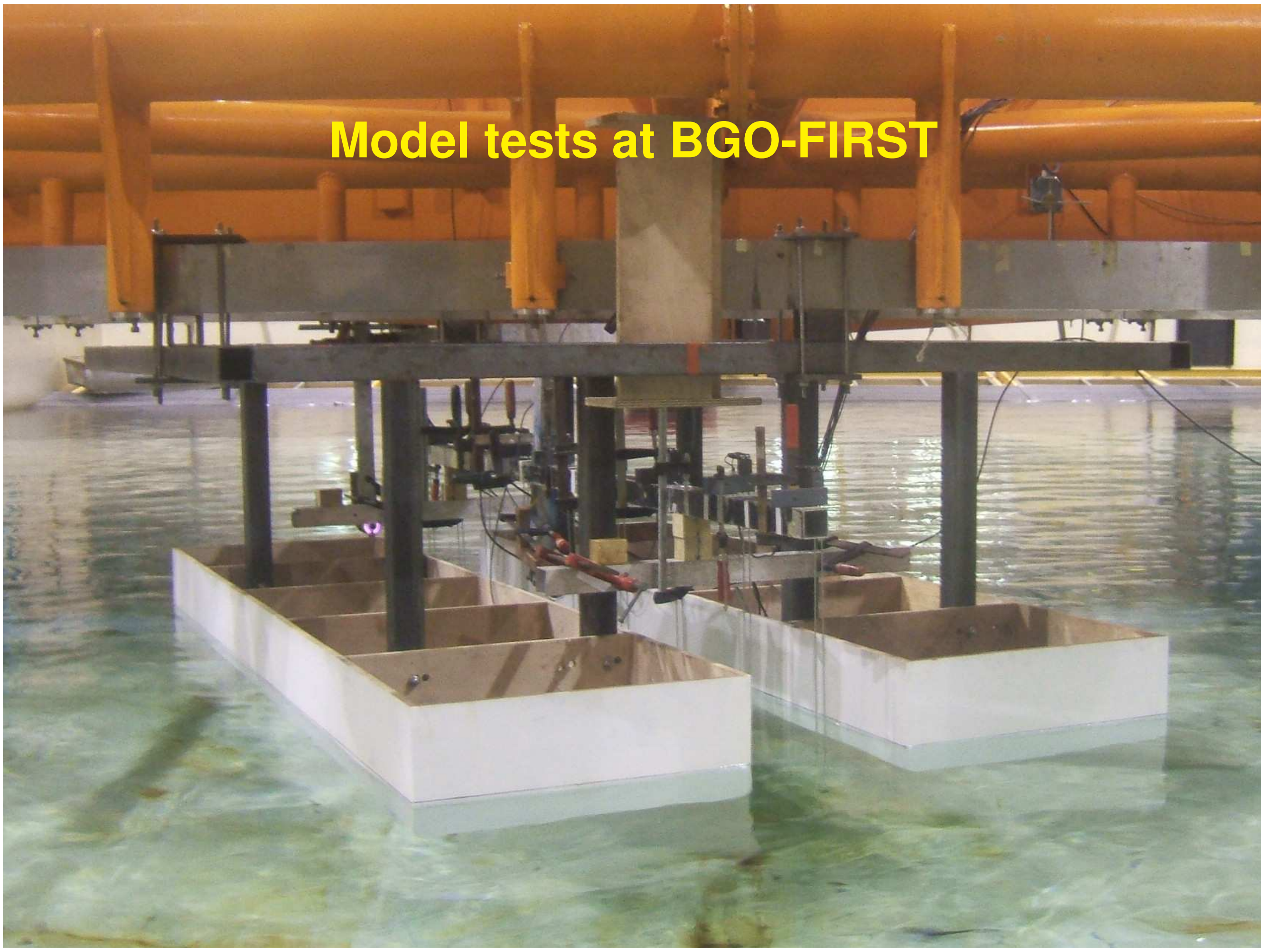
Saipem's concepts of WHB (Well-Head Barge) and MFB (Multi-Function Barge)



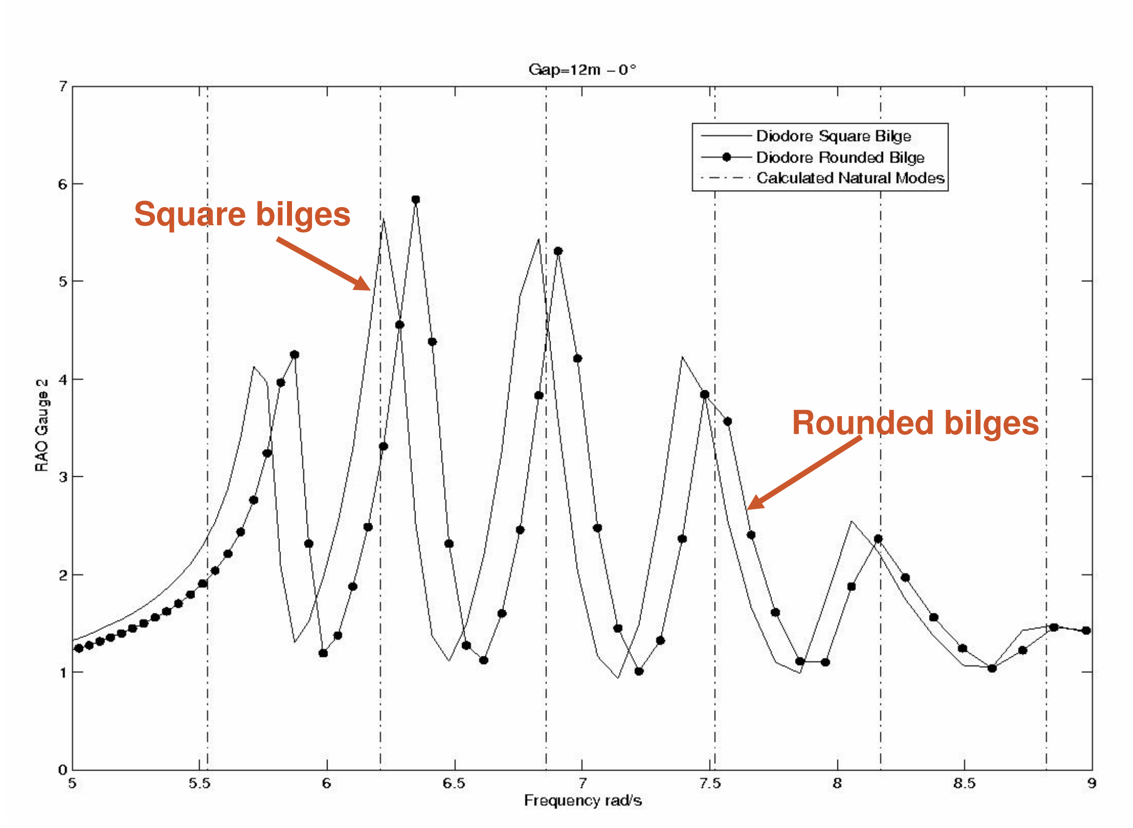
Ideol's floating foundation for wind turbines.
The « damping pool ».



Prelude turret



Heading 0°. Calculated Free Surface RAOs at gauge 2



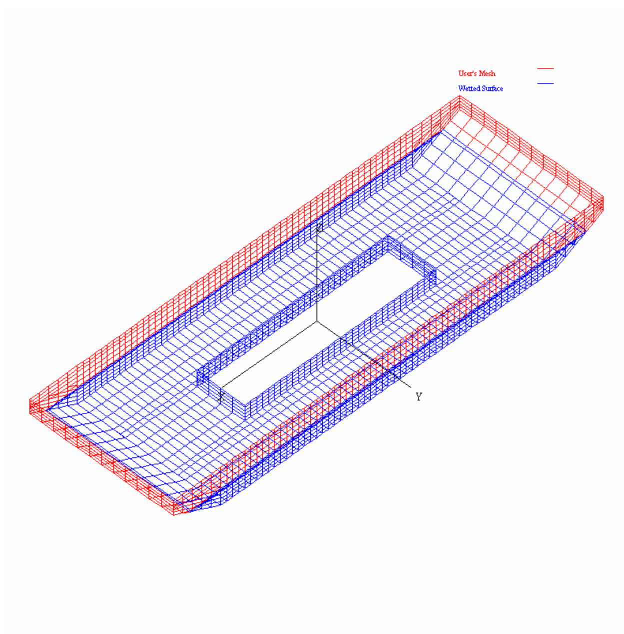
Resonant modes in moonpools

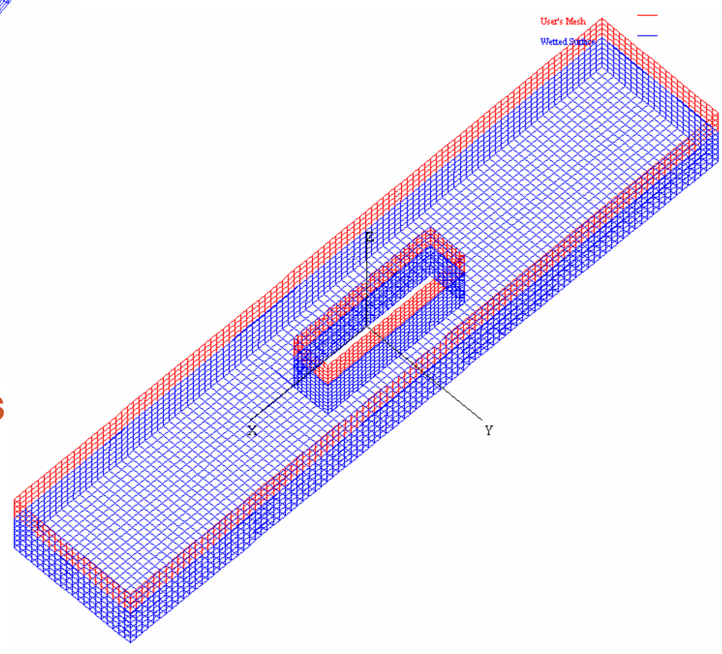
- Piston mode (up and down motion)
- Sloshing modes (alike in a tank)

How to (simply) predict the natural frequencies and associated modal shapes of the free surface?

1. Rectangular moonpools / gaps in deep water

Meshes for Diodore calculations

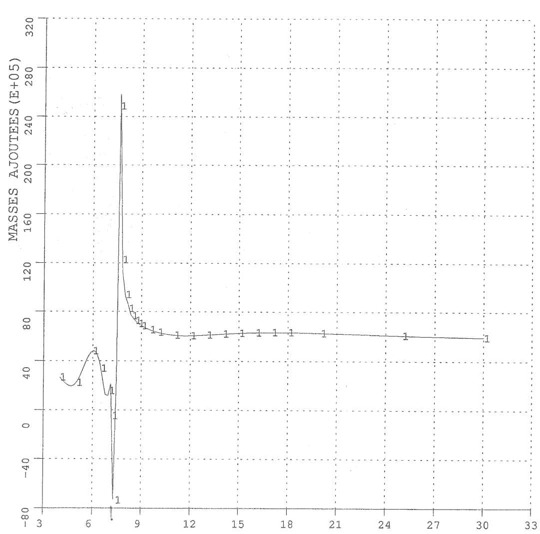




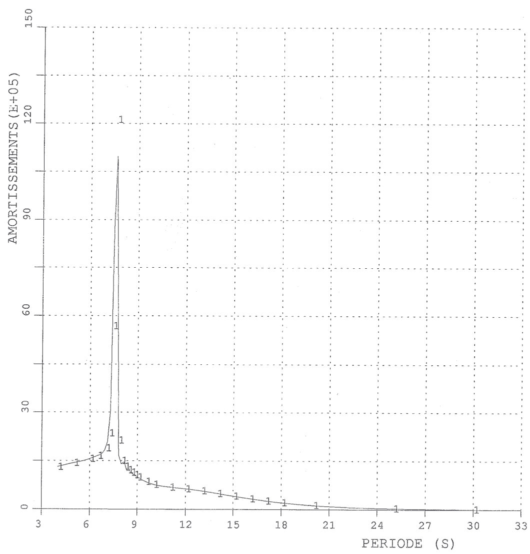
Moonpool dimensions
80 m x 20 m

Moonpool dimensions: 80 m x 20 m x 6.5 m

First sloshing mode: T = 7.5 s

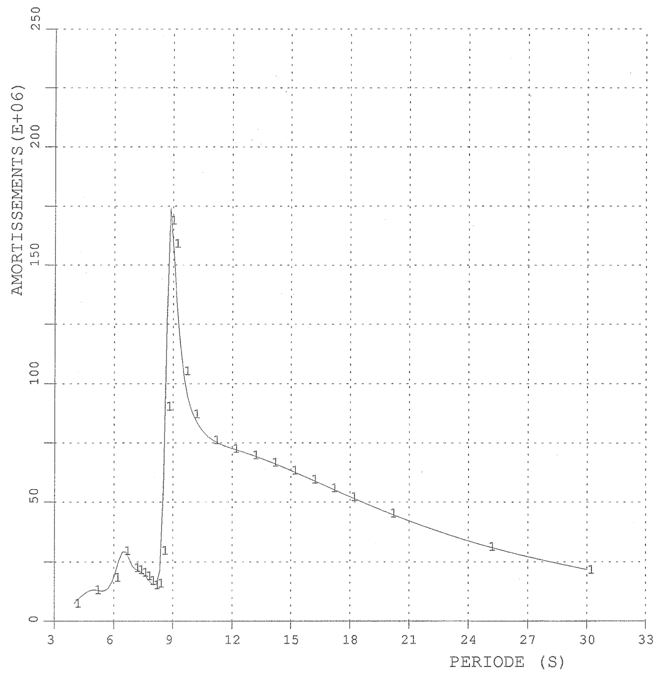


Surge added mass



Surge radiation damping

Heave radiation damping



Piston mode: T = 9 s

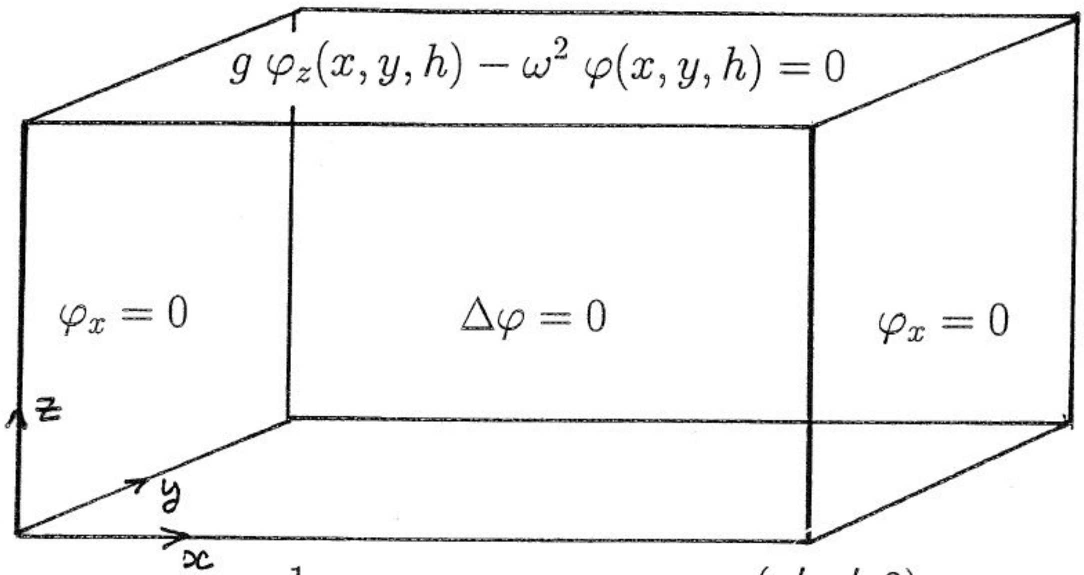
Theoretical model

Main assumptions:

- Rectangular moonpool, vertical walls from deck to keel
- Motionless support
- No outer free surface (length and beam taken infinite)
- Infinite waterdepth
- Linearized potential flow theory

(Molin, On the piston and sloshing modes in moonpools, *J. Fluid Mech.*, 2001)

→ Boundary Value Problem



$$\varphi(x,y,0) = \frac{1}{2\pi}\int_0^b dy' \int_0^l dx' \, \frac{\varphi_z(x',y',0)}{\sqrt{(x-x')^2+(y-y')^2}}$$

Theoretical frame: linearized potential flow theory.

Velocity potential in the upper domain (moonpool):

$$\Phi^+(x,y,z,t) = \varphi(x,y,z)\ \cos(\omega\, t + \theta)$$

General expression for the velocity potential inside the moonpool:

$$\varphi(x,y,z) = \sum_{n=0}^{N}\sum_{q=0}^{Q} \cos\lambda_n\, x\ \cos\mu_q\, y\ (A_{nq}\ \cosh\nu_{nq}\, z + B_{nq}\ \sinh\nu_{nq}\, z)$$

where $\lambda_n = n\,\pi/l$, $\mu_q = q\,\pi/b$, $\nu_{nq}^2 = \lambda_n^2 + \mu_q^2$
and, when $n = q = 0$, the hyperbolic functions are replaced with $A_{00} + B_{00}\ z/h$.

Verifies the Laplace equation and the no-flow conditions at the vertical walls.

Remaining conditions to be fulfilled:

- Free surface condition

$$g\,\varphi_z - \omega^2\,\varphi = 0$$

- Matching condition with the flow in the lower domain:

$$\varphi(x,y,0,t) = \frac{1}{2\,\pi}\int_0^l \mathrm{d}x' \int_0^b \mathrm{d}y'\, \frac{\varphi_z(x',y',0,t)}{\sqrt{(x-x')^2+(y-y')^2}}$$

Galerkin procedure: insert the general expression of φ into the bottom (resp. free surface) boundary condition, multiply each side with $\cos\lambda_m x\ \cos\mu_p y$ and integrate in x and y over the bottom (resp. free surface).

Requires such integrals as

$$I_{mnpq} = \int_0^l \mathrm{d}x \int_0^l \mathrm{d}x' \int_0^b \mathrm{d}y \int_0^b \mathrm{d}y'\, \frac{\cos\lambda_m x\ \cos\lambda_n x'\ \cos\mu_p y\ \cos\mu_q y'}{\sqrt{(x-x')^2+(y-y')^2}}$$

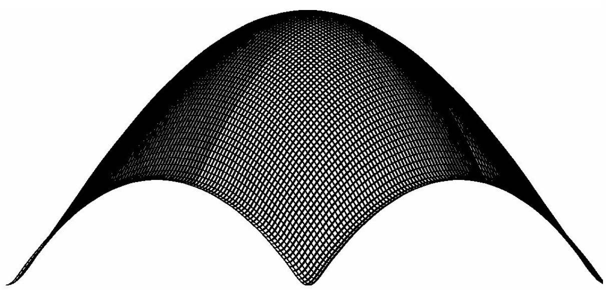
to be evaluated

Eliminating A_{mp} between free surface and bottom boundary conditions gives an eigen value problem in the form

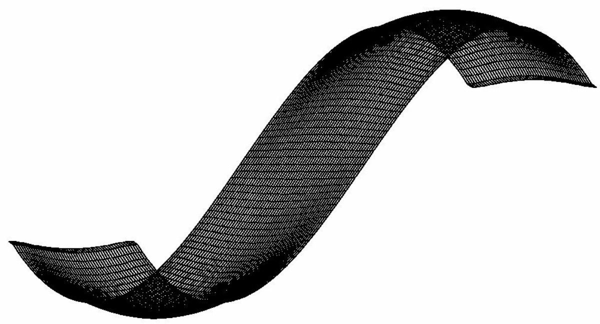
$$\mathrm{M}_1\,\vec{B} = \omega^2\,\mathrm{M}_2\,\vec{B}$$

with $\vec{B} = (B_{00}, B_{01}, \ldots, B_{mp}, \ldots)$

Resolution $\rightarrow$ eigen-frequencies ω_{mp} and modal shapes.



Piston mode 40 x 40 x 1



First sloshing mode 80 x 20 x 1

Single mode approximations

Piston mode:

$$\omega_{00}^2 \simeq \frac{g}{h + b\ f_3(b/l)}$$

where

$$f_3 \;=\; \frac{1}{\pi}\left\{\arg\sinh\frac{l}{b} + \frac{l}{b}\ \arg\sinh\frac{b}{l} + \frac{1}{3}\left(\frac{b}{l} + \frac{l^2}{b^2}\right)\right.$$
$$\left. -\frac{1}{3}\left(1 + \frac{l^2}{b^2}\right)\ \sqrt{\frac{b^2}{l^2} + 1}\ \right\}$$

Longitudinal sloshing modes:

$$\omega_{n0}^2 \simeq g\,\lambda_n\,\frac{1 + J_{n0}\ \tanh\lambda_n h}{J_{n0} + \tanh\lambda_n h}$$

where

$$J_{n0} \;=\; \frac{2}{n\,\pi^2\,r}\left\{\int_0^1 \frac{r^2}{u^2\,\sqrt{u^2+r^2}}\ \left[1 + (u-1)\ \cos(n\,\pi\,u) - \frac{\sin(n\,\pi\,u)}{n\,\pi}\right]\ du\right.$$
$$\left. + \frac{1}{\sin\theta_0} - 1\right\}$$

$$(\lambda_n = n\,\pi/l \qquad r = b/l \qquad \tan\theta_0 = r^{-1})$$

Single mode approximation for gaps:

Replace Neumann condition $\partial\varphi/\partial x = 0$ with Dirichlet condition $\varphi = 0$ at moonpool ends $x = 0,\ x = l$.

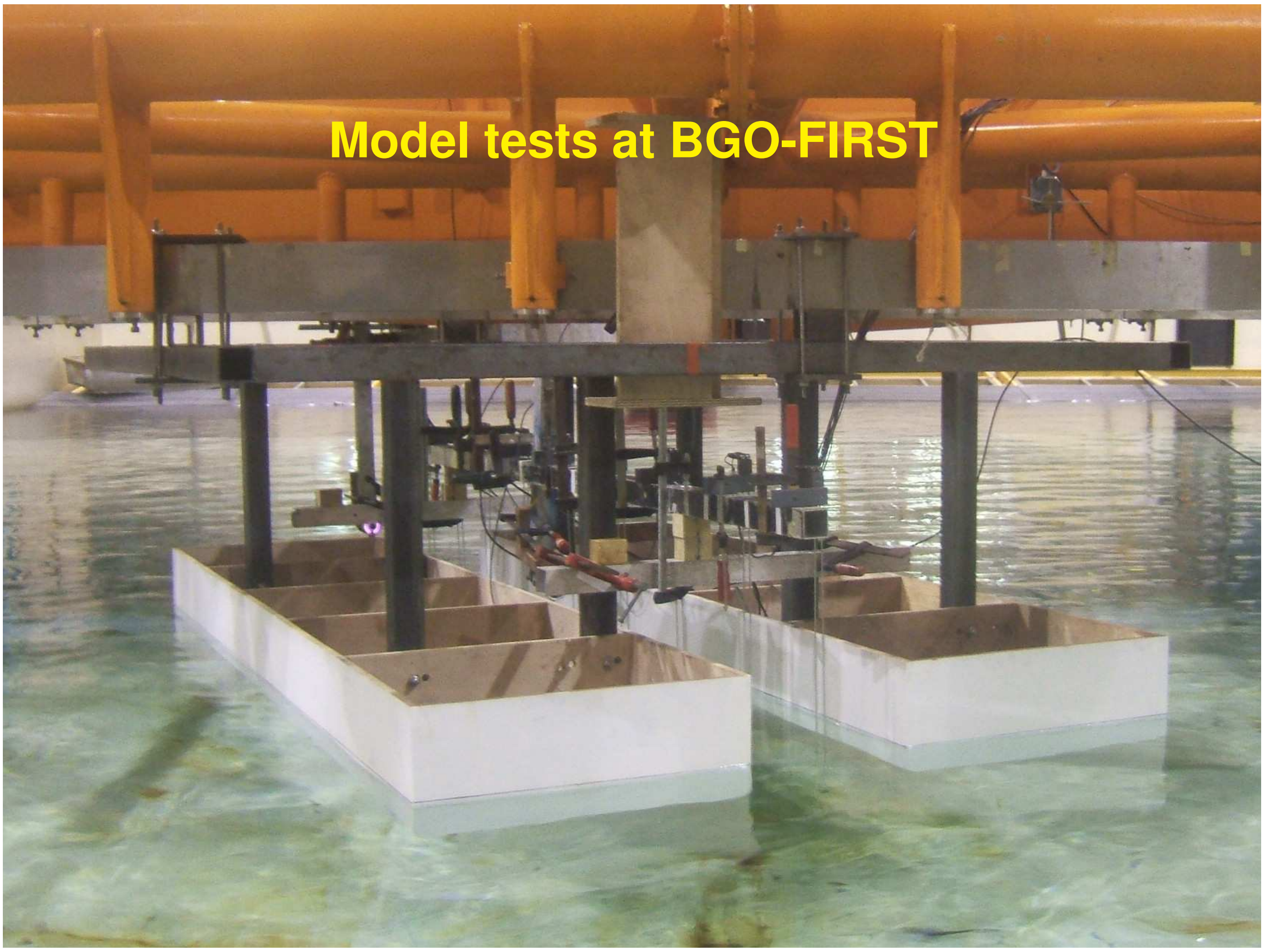
$\rightarrow \cos\lambda_n x$ changed into $\sin\lambda_n x$

$$\omega_{n0}^2 \simeq g\,\lambda_n\,\frac{1 + J_{Dn0}\ \tanh\lambda_n h}{J_{Dn0} + \tanh\lambda_n h}$$

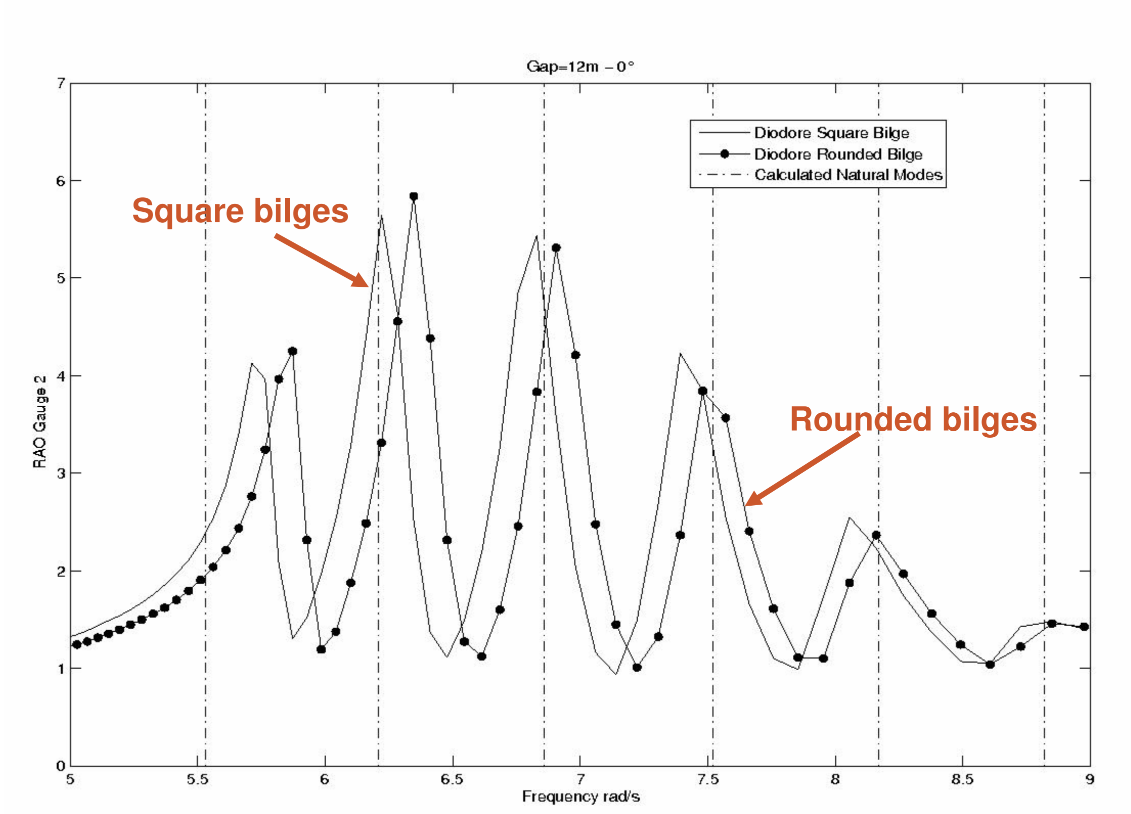
where

$$J_{Dn0}(r) \;=\; \frac{2}{n\,\pi^2\,r}\left\{\int_0^1 \frac{r^2}{u^2\,\sqrt{u^2+r^2}}\,[1 + 2u\right.$$
$$+ (u-1)\ \cos(n\,\pi\,u) - \frac{3}{n\,\pi}\ \sin(n\,\pi\,u)]\ du$$
$$\left. - \frac{1}{\sin\theta_0} + 1 + 2r\ \ln\frac{1+\cos\theta_0}{1-\cos\theta_0}\right\}$$

(Molin *et al.*, APOR 2002)



Heading 0°. Calculated Free Surface RAOs at gauge 2



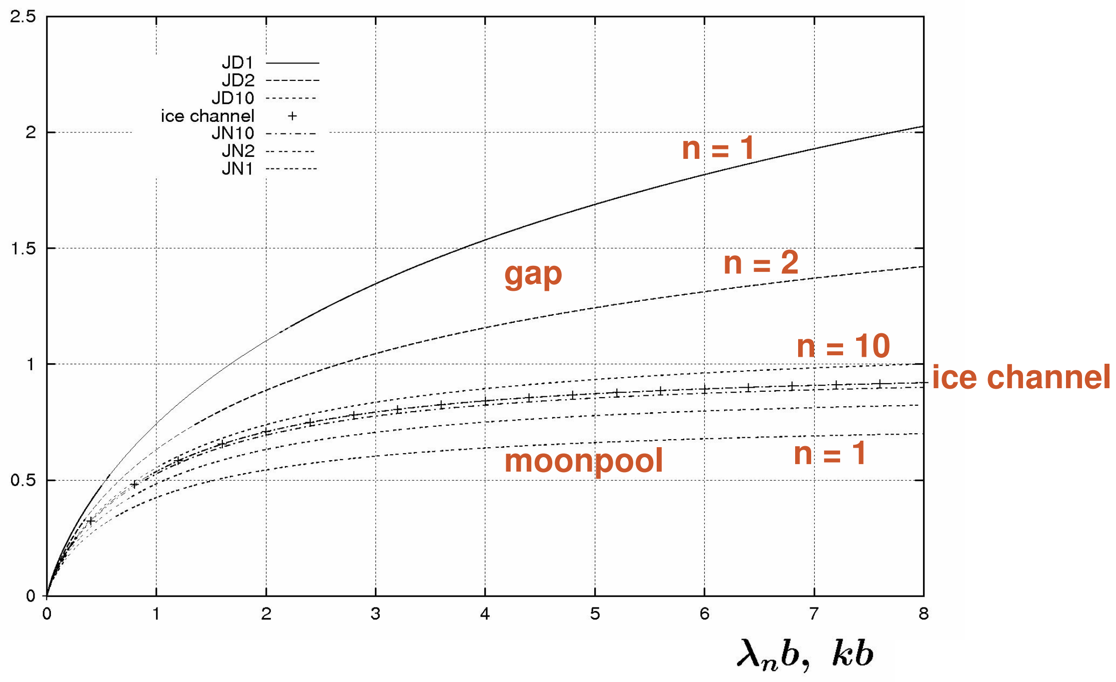
Ice channel

$$\omega^2 \simeq g\,k\,\frac{1+\widetilde{J}_0\ \tanh kh}{\widetilde{J}_0+\tanh kh}$$

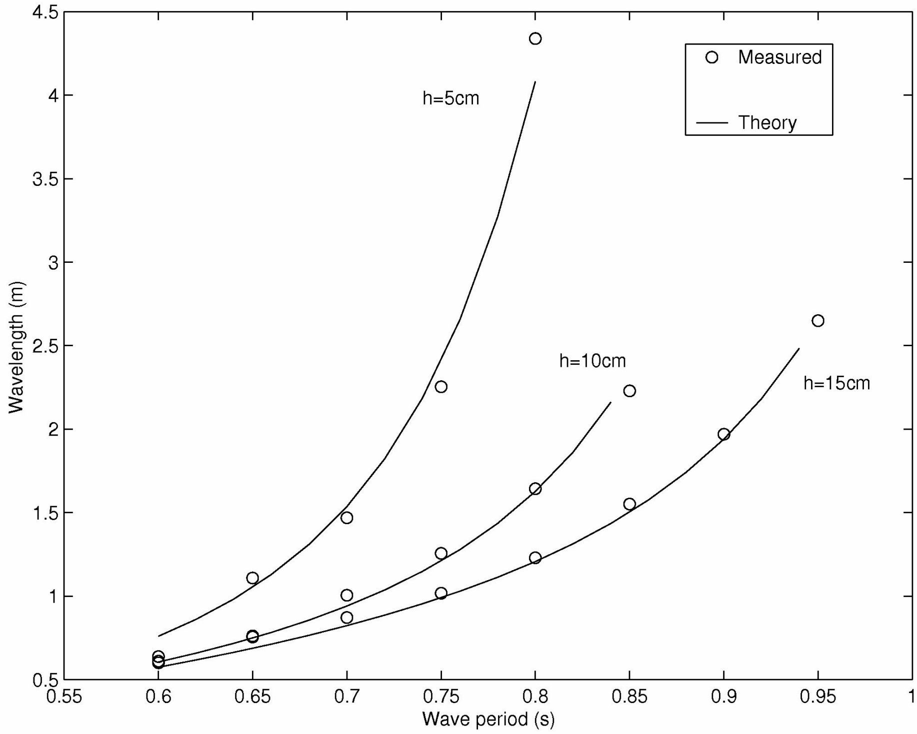
where

$$\begin{aligned}\widetilde{J}_0(kb) &= 1-\frac{2}{\pi\,kb}\left(1-\int_0^1 \mathrm{e}^{-kb/\sqrt{1-u^2}}\,\mathrm{d}u\right)\\ &\simeq \frac{kb}{\pi}\left(\ln\frac{2}{kb}+\frac{3}{2}-\gamma\right)\end{aligned}$$

(for $kb < 0.5$)



Ice channel: verification of the dispersion equation

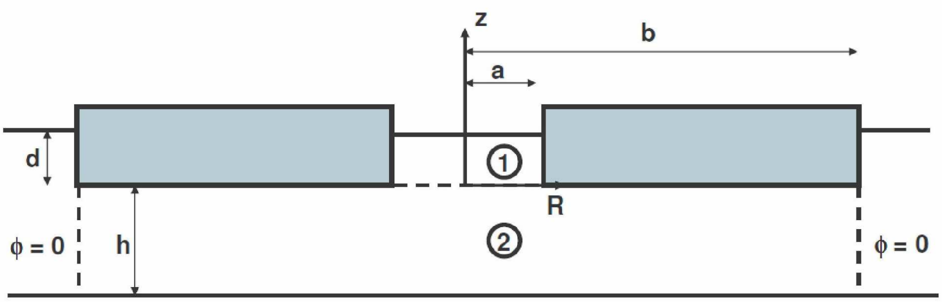


(Molin *et al.*, APOR 2002)

2. Moonpools and gaps in finite depth

(In collaboration with Xinshu Zhang, SJTU)

Circular moonpool in finite depth



Use eigen-function expansions:

$$\varphi_1(R,z) = A_0 + B_0\,\frac{z}{d} + \sum_{n=1}^{\infty}(A_n\,\cosh k_n z + B_n\,\sinh k_n z)\;J_0(k_n R)$$

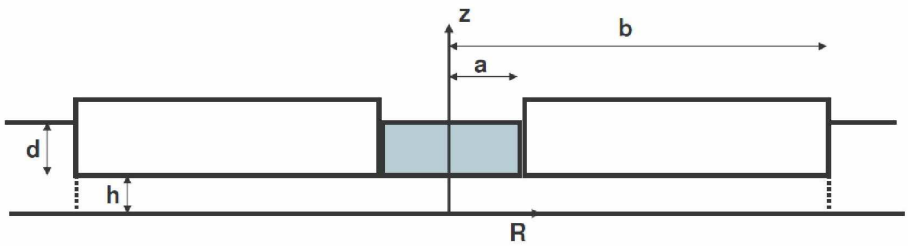
$$\varphi_2(R,z) = \sum_{n=1}^{\infty} C_n\,\frac{\cosh\lambda_n(z+h)}{\cosh\lambda_n h}\,J_0(\lambda_n R)$$

with k_n the roots of $J_0'(k_n a) = 0$ and λ_n the roots of $J_0(\lambda_n b) = 0$

and apply Garrett's method.

(Garrett CJR Waves forces on a circular dock, JFM 1971)

Frozen approximation

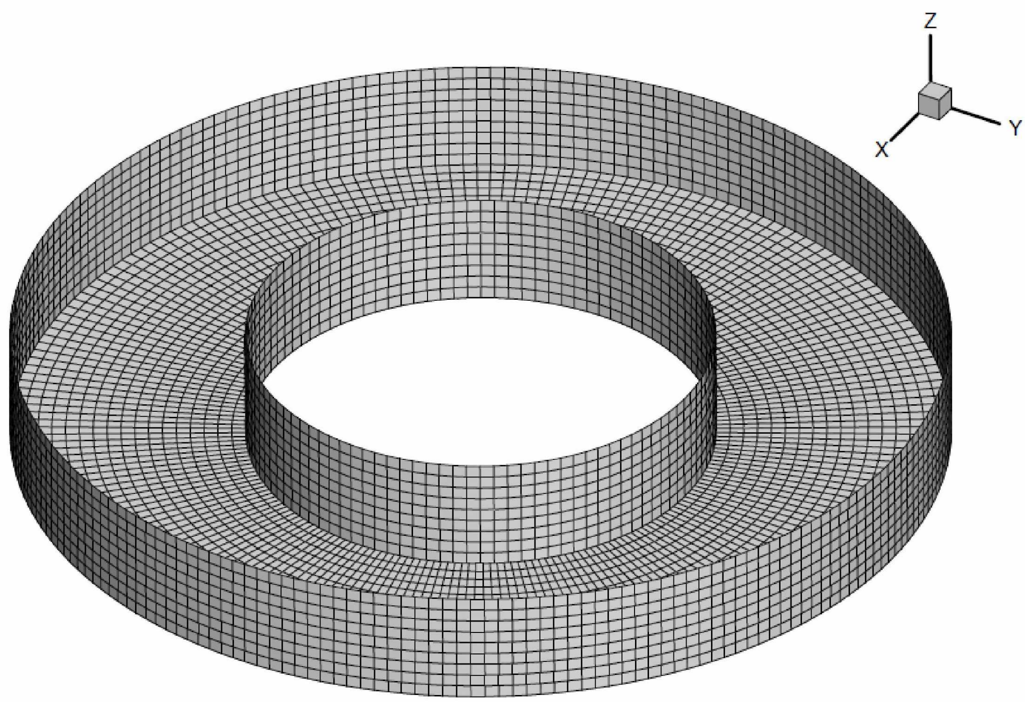


Vertical added mass obtained as

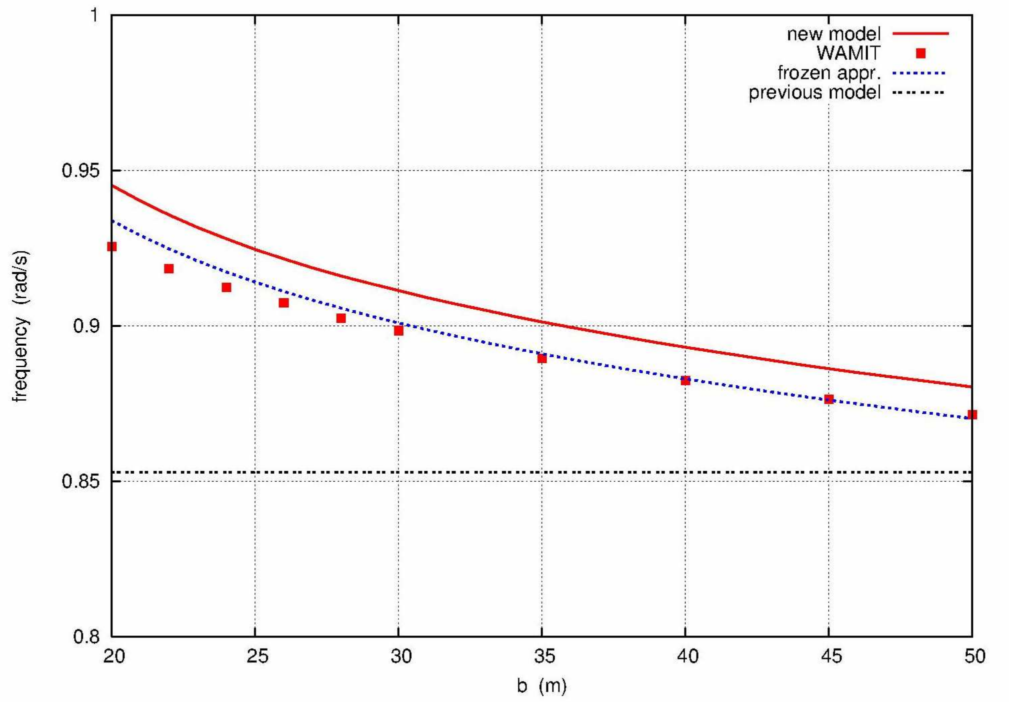
$$M_{33} = 4\,\rho\,\pi\,\frac{a^2}{b^2}\sum_{n=1}^{\infty}\frac{J_1^2(\lambda_n a)}{\lambda_n^3\,\tanh\lambda_n h\;J_1^2(\lambda_n b)}$$

When $h \to \infty$ and $b \to \infty$ $\qquad M_{33} \to 8/3\,\rho\,a^3$

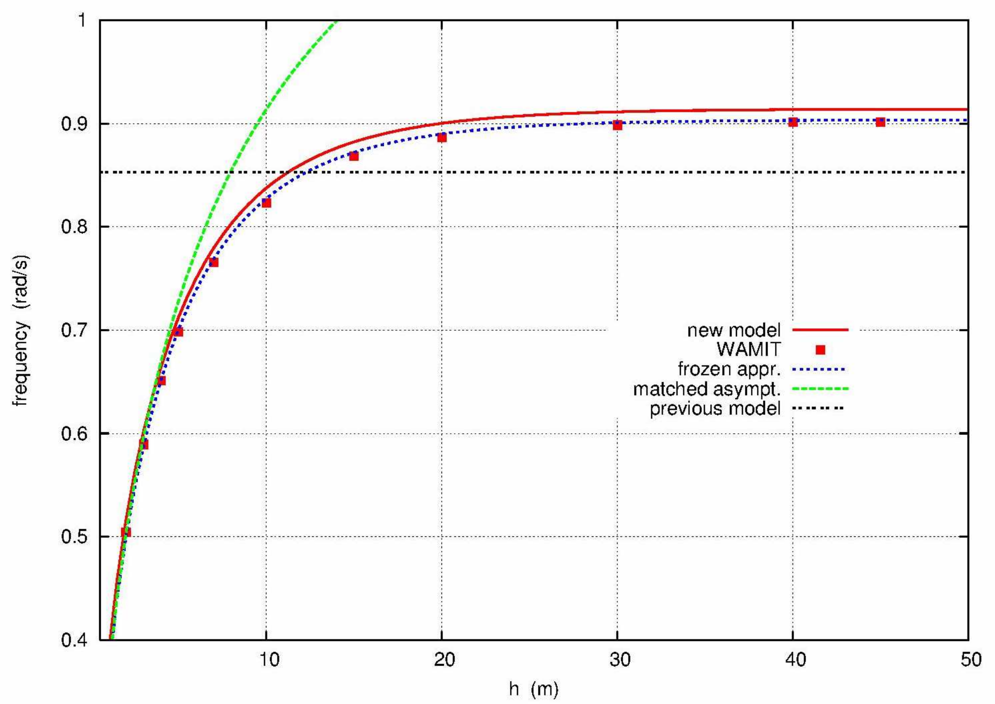
When $h \to 0$ $\qquad M_{33} \simeq \rho\,\pi\,a^4/(8\,h)\;[1 + 4\,\ln(b/a)]$



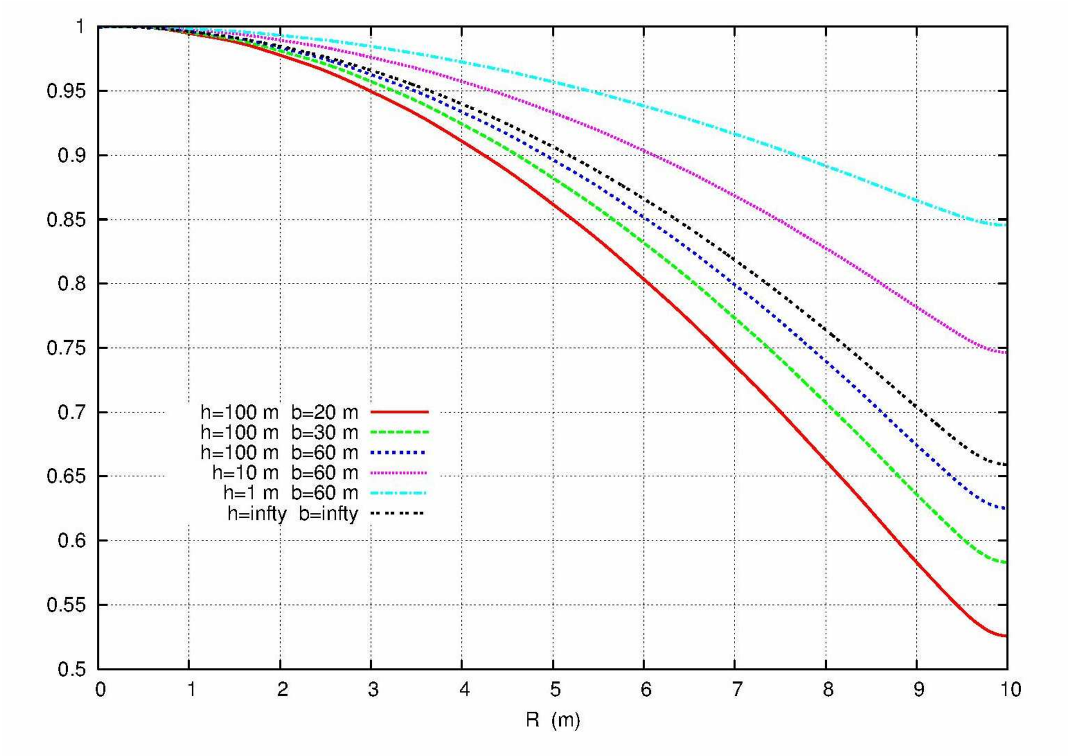
Mesh for WAMIT calculations (SJTU)



Natural frequency of the piston mode vs outer radius. Inner radius : 10 m. Draft : 5 m. Clearance : 30 m

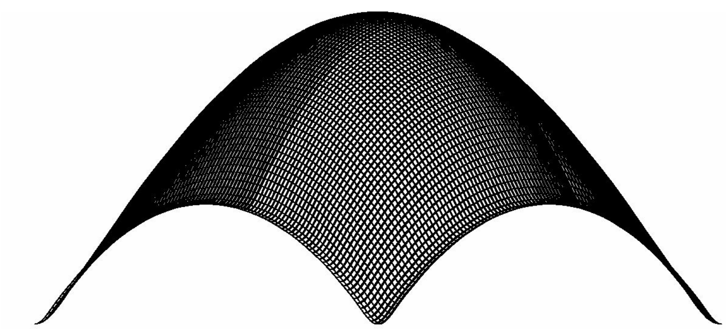


Natural frequency of the piston mode vs clearance. Inner radius : 10 m. Outer radius : 30 m. Draft : 5 m.



Modal shape of the piston mode.
Inner radius 10 m; draft 0.443 m.

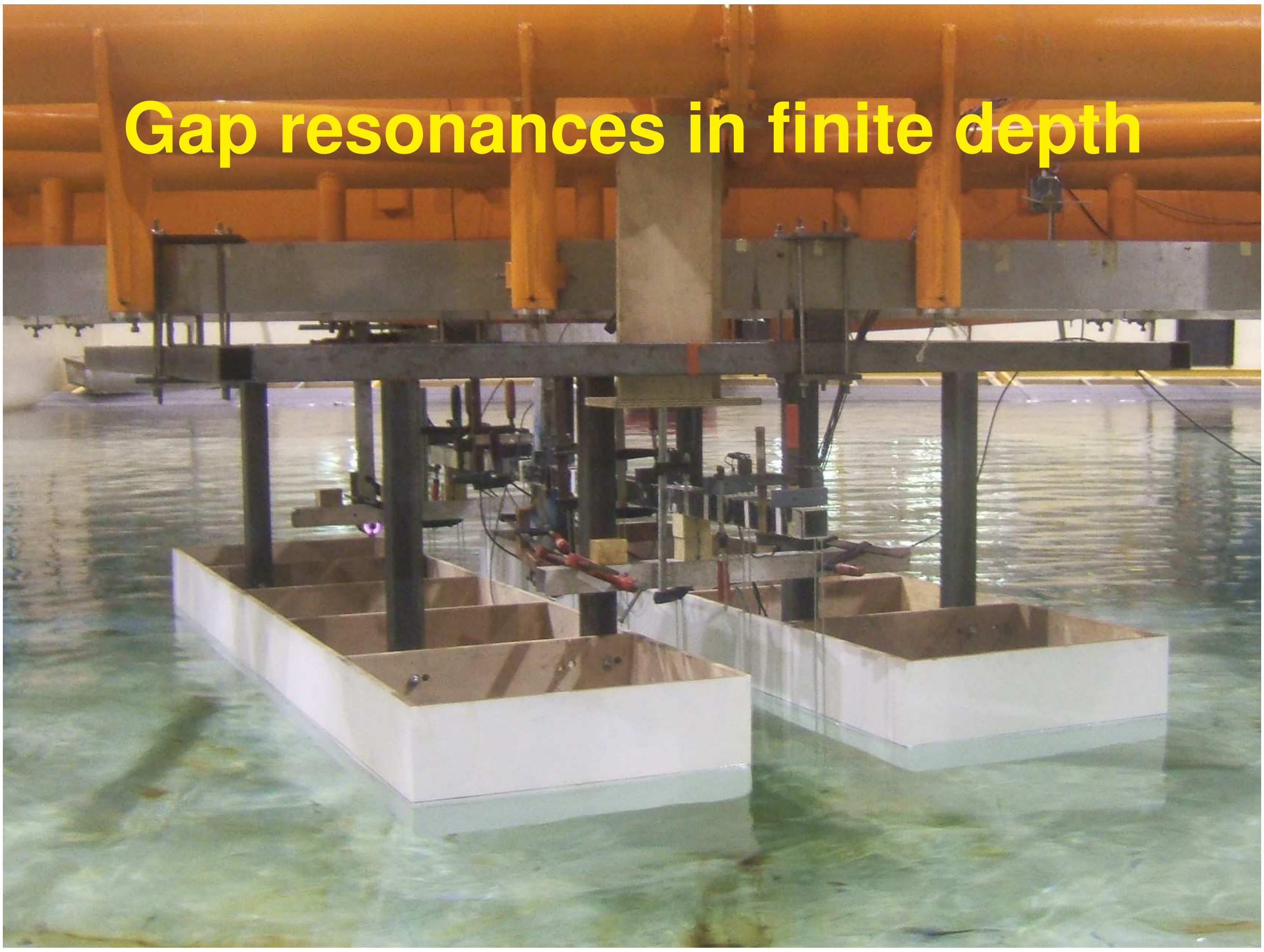
Square case

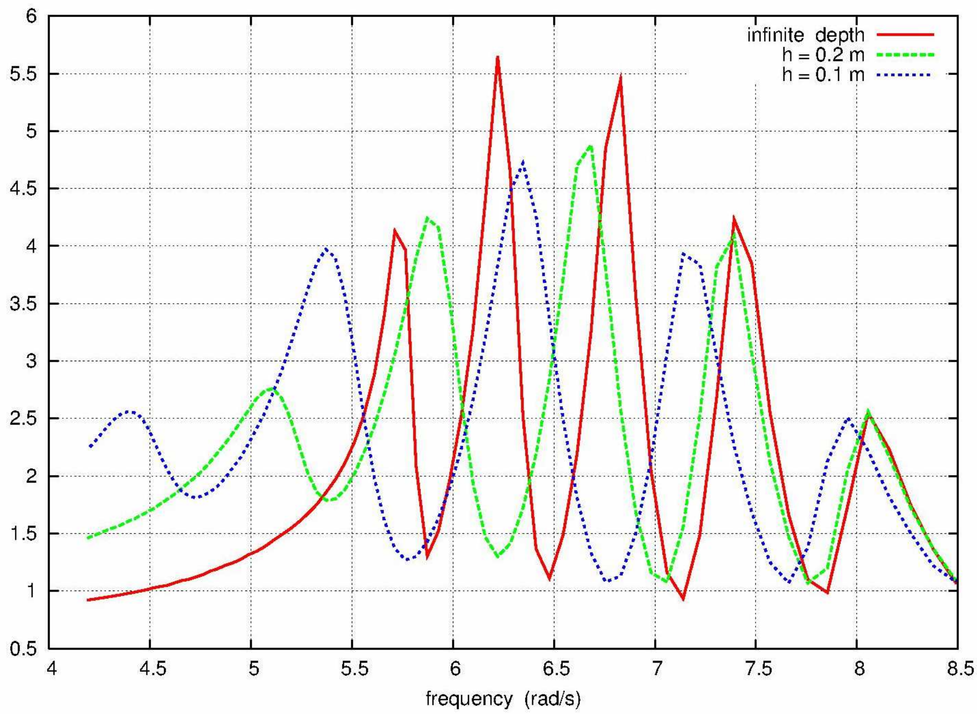


Piston mode 40 x 40 x 1

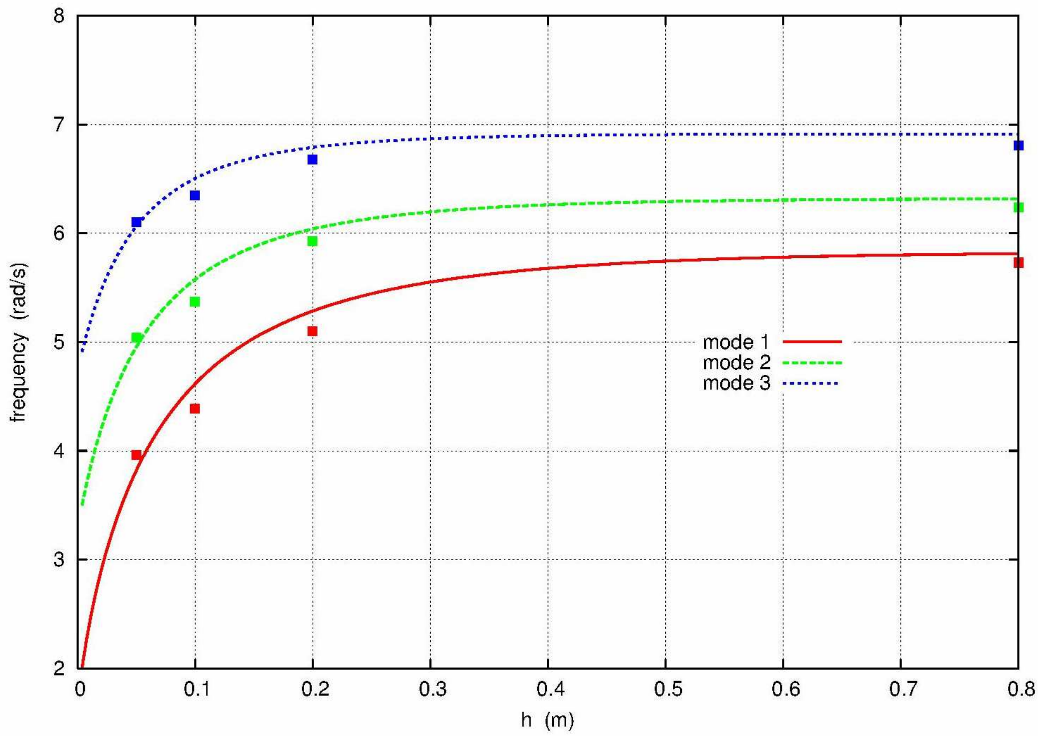
(0.54 in the corner)

Gap resonances in finite depth





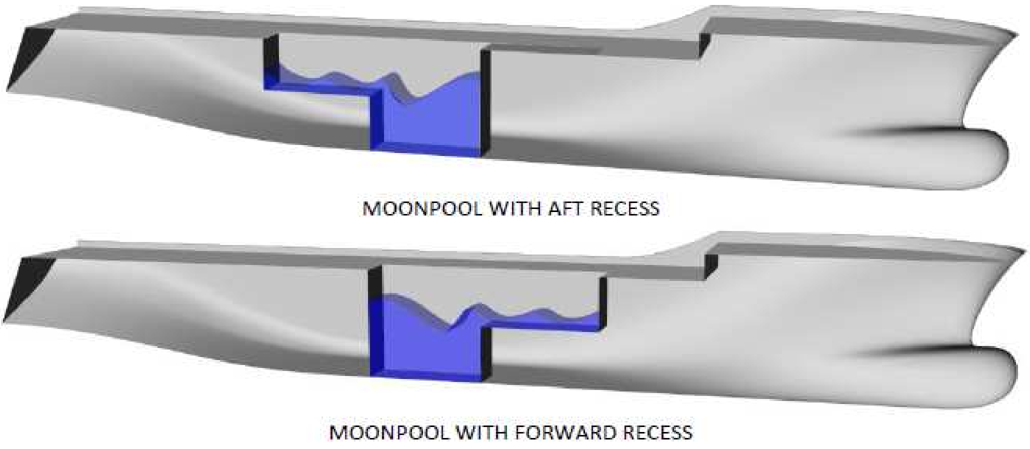
Diodore calculations



3. Rectangular moonpools with recesses

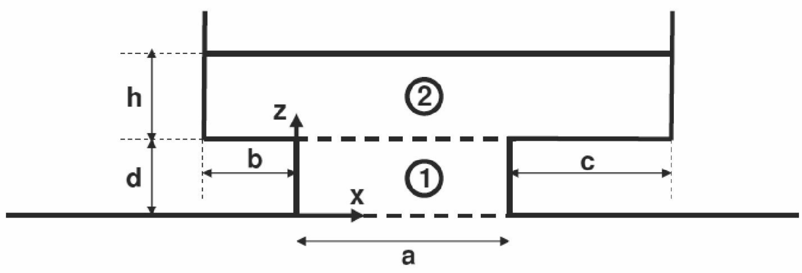
(in deep water)

Moonpools with recesses



(Hammargren & Törnblom, 2012)

Rectangular moonpool with recesses



Flow assumed to be two-dimensional in the moonpool.

Eigen-function expansions:

$$\varphi_1(x,y,z) = A_0 + B_0 \frac{z}{d} + \sum_{n=1}^{\infty} (A_n \cosh k_n z + B_n \sinh k_n z) \cos k_n x$$

$$\varphi_2(x,y,z) = C_0 + D_0 \frac{z-d}{h} + \sum_{n=1}^{\infty} (C_n \cosh \lambda_n (z-d) + D_n \sinh \lambda_n (z-d)) \cos \lambda_n (x+b)$$

with $k_n = n\pi/a$ and $\lambda_n = n\pi/(a+b+c)$.

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Study on Hydrodynamic Performances of a Deep-Water Drillship and Water Motions inside Its Rectangular Moonpool

Xiaoxian Guo, Haining Lu, Jianmin Yang, Tao Peng
State Key Laboratory of Ocean Engineering, Shanghai Jiao Tong University, Shanghai, China.
Collaborative Innovation Center for Advanced Ship and Deep-Sea Exploration(CISSE), Shanghai, China.

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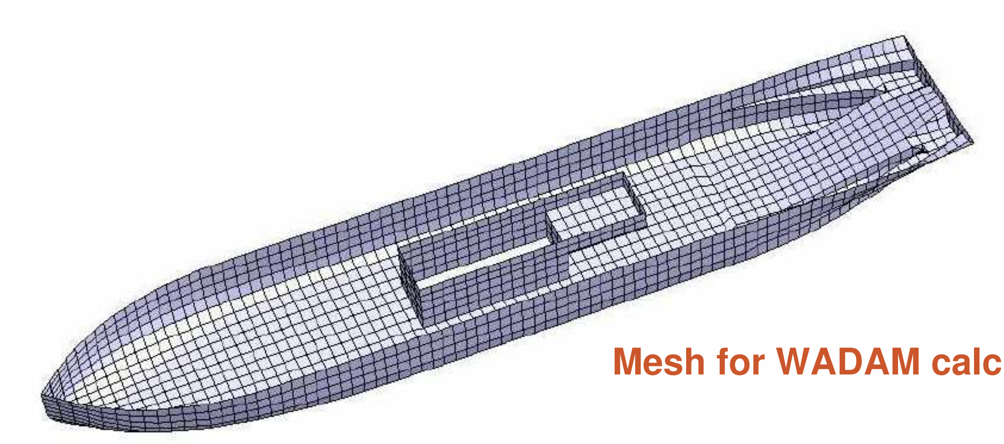
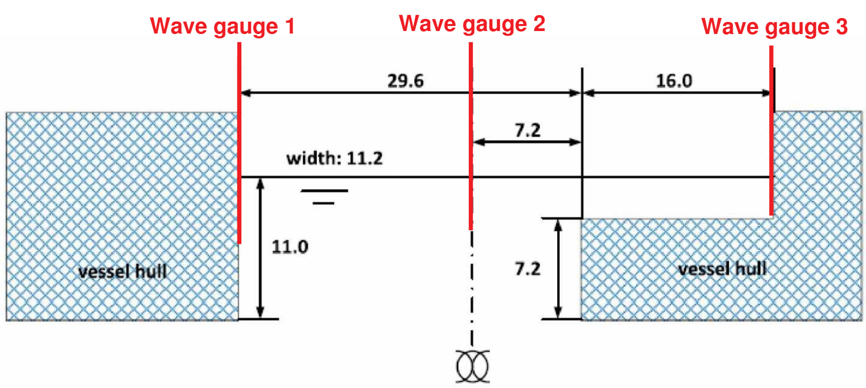
Resonant water motions within a recessing type moonpool in a drilling vessel

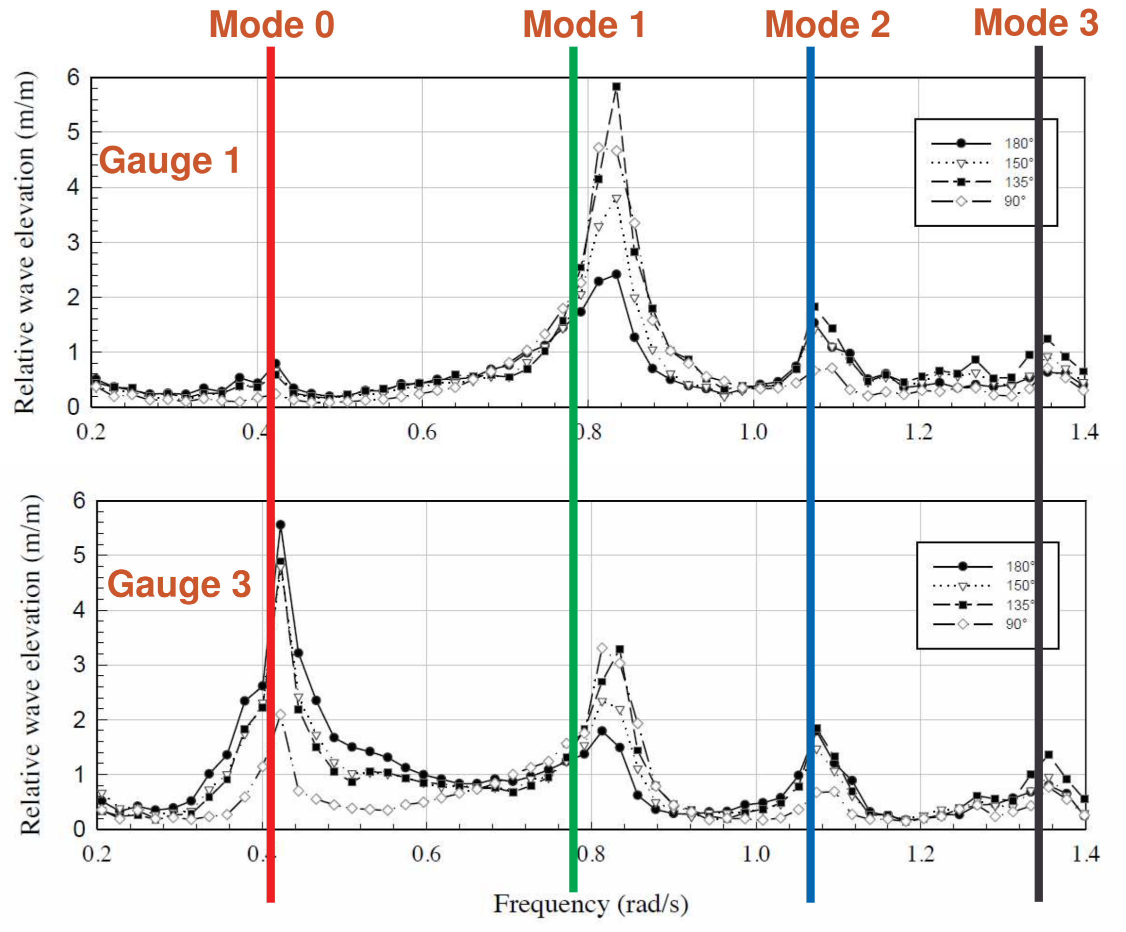
Xiaoxian Guo[a,b], Haining Lu[a,b,*], Jianmin Yang[a,b], Tao Peng[a,b]

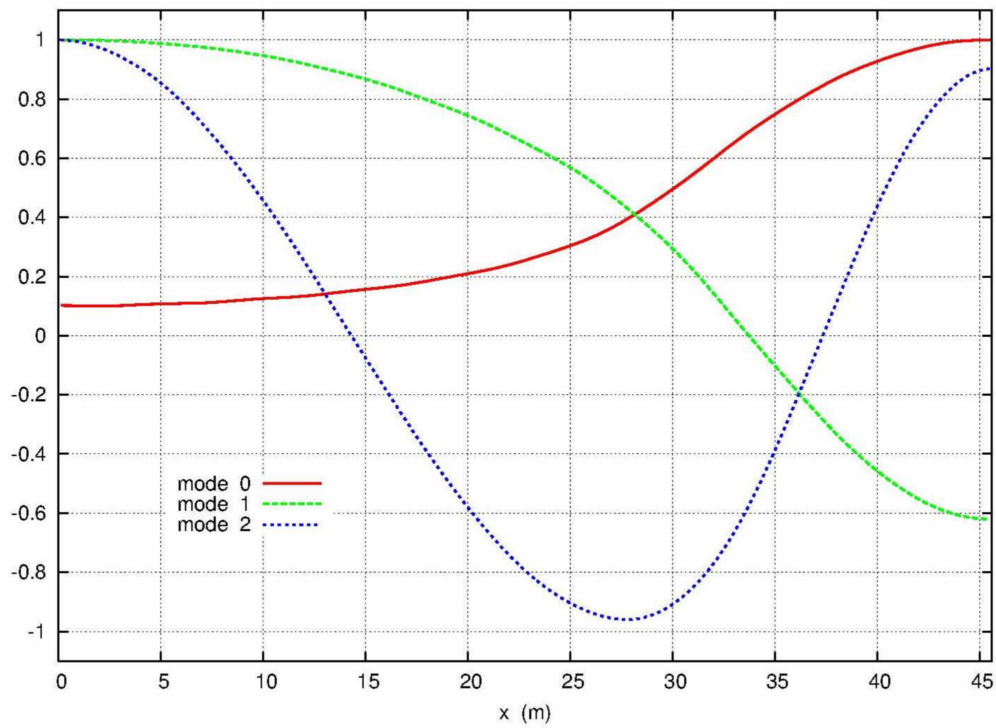
[a] *State Key Laboratory of Ocean Engineering, Shanghai Jiao Tong University, Shanghai 200240, China*
[b] *Collaborative Innovation Center for Advanced Ship and Deep-Sea Exploration (CISSE), Shanghai 200240, China*



Mesh for WADAM calculations

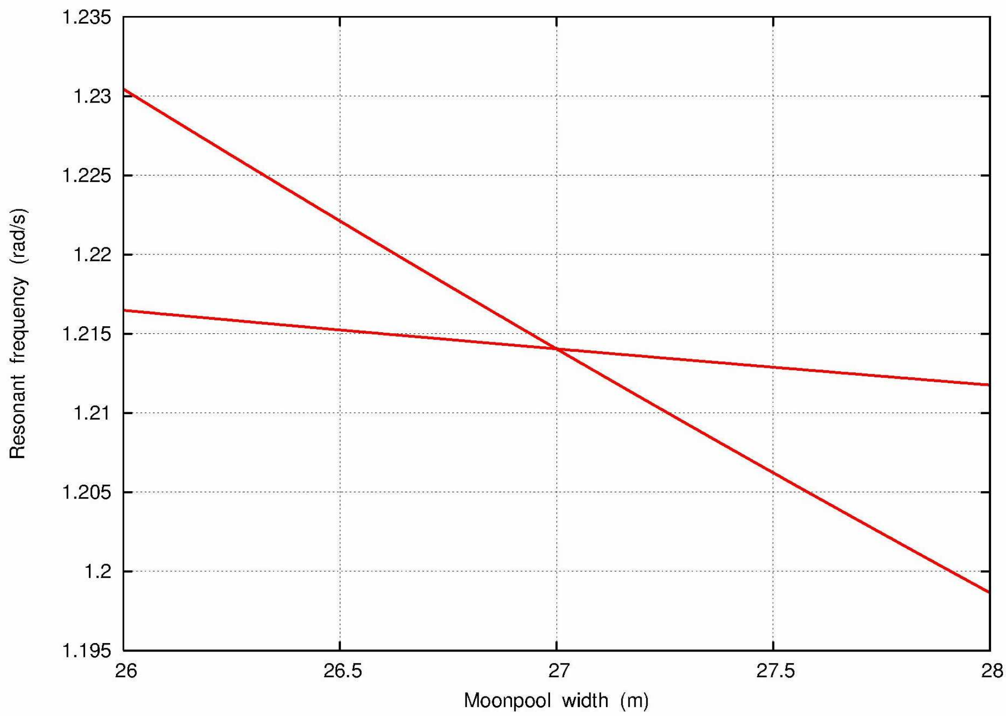




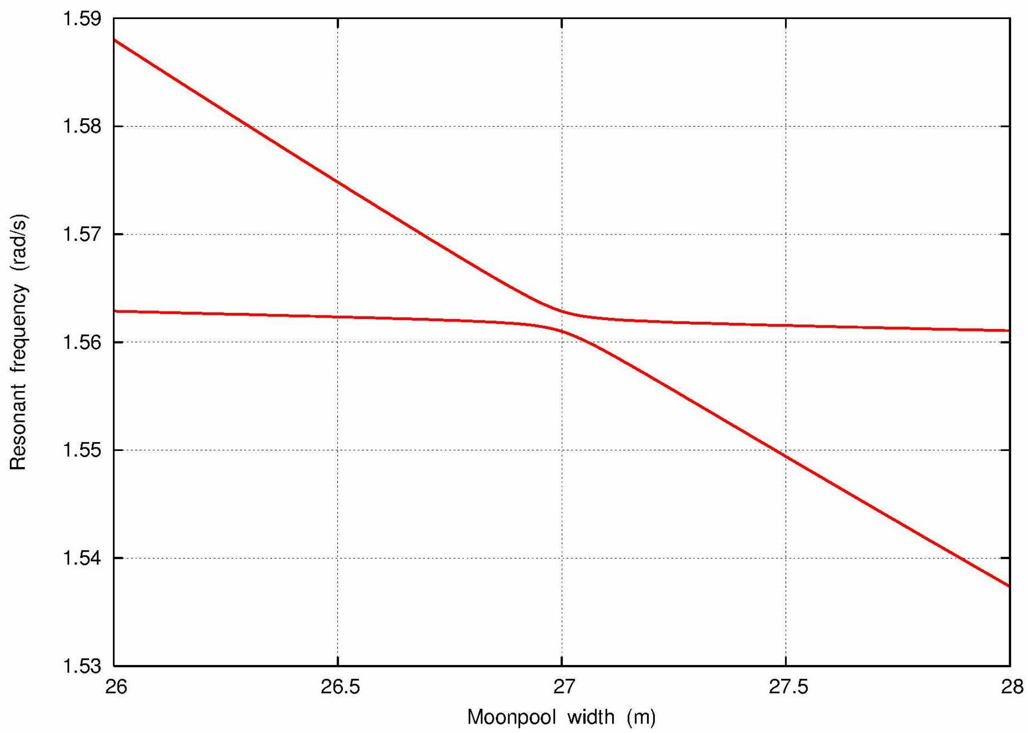


Calculated modal shapes

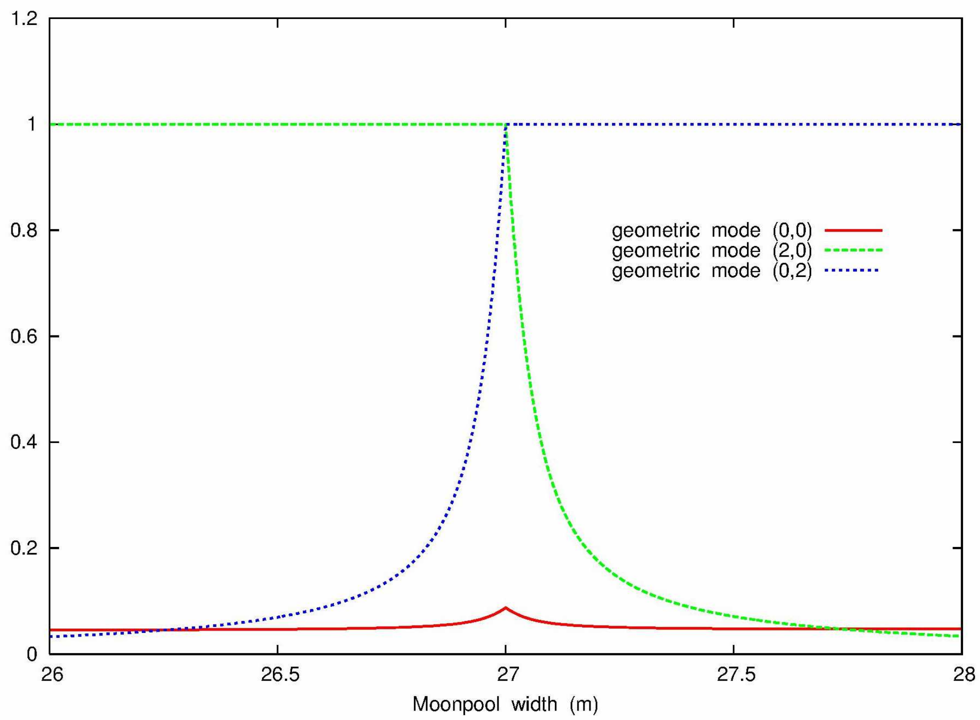
Sloshing modes in square or nearly square moonpools



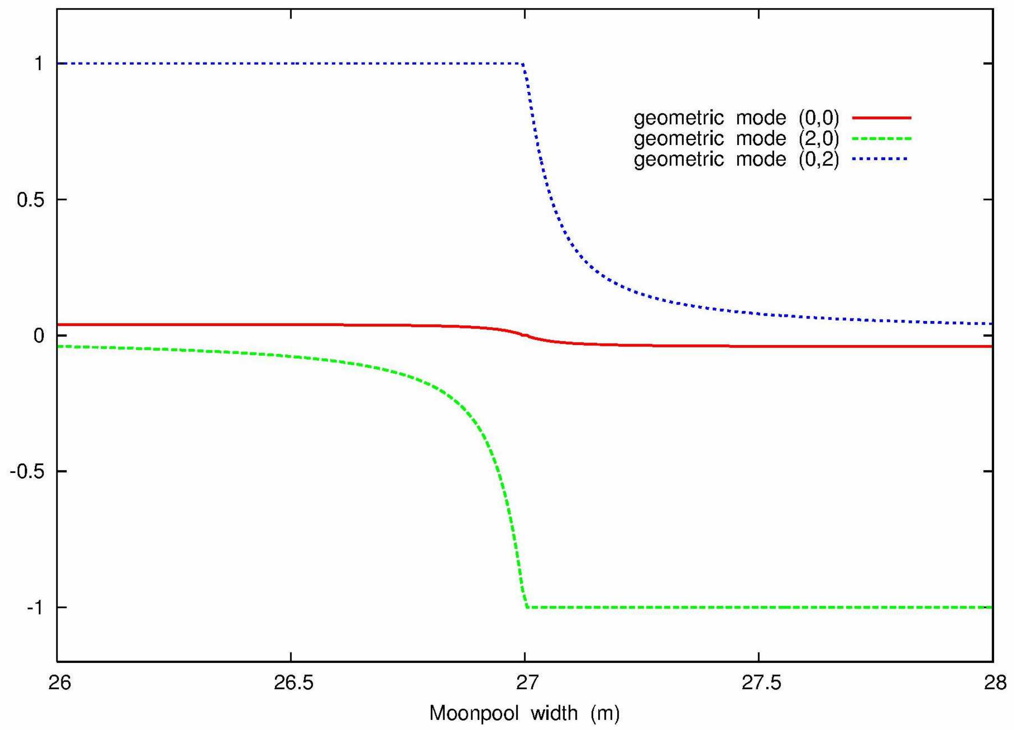
First sloshing modes. Resonant frequencies vs width.



Second sloshing modes. Resonant frequencies vs width.



$$\eta(x,y,t) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} A_{mn} \cos \lambda_m x \cos \mu_n y \cos \omega t$$



$$\eta(x,y,t) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} A_{mn} \; \cos\lambda_m x \; \cos\mu_n y \; \cos\omega t$$

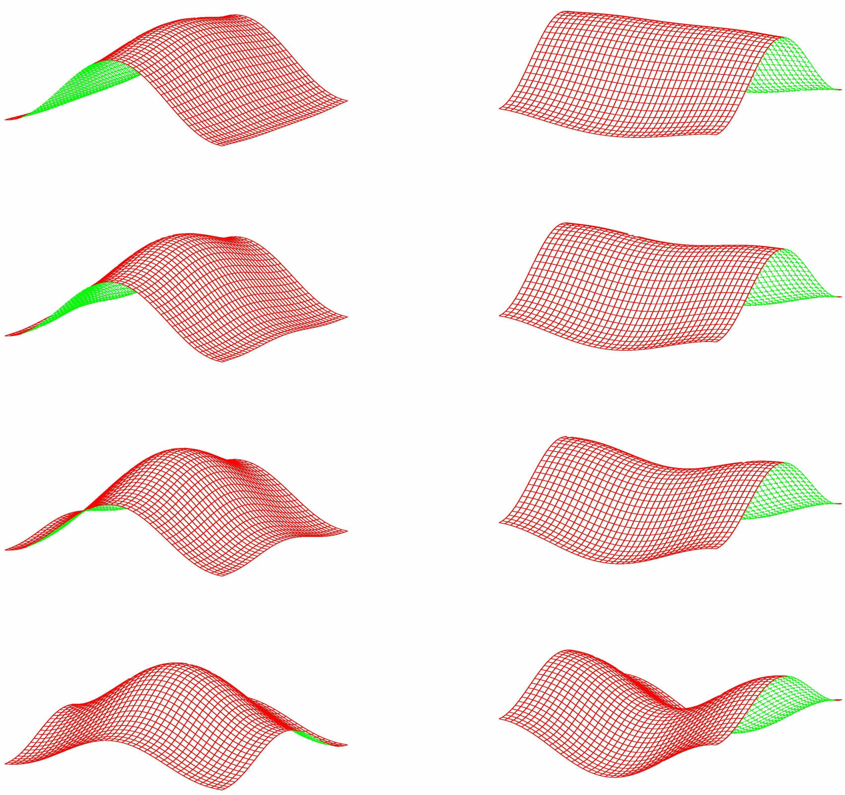
Width

26.5 m

26.8 m

26.9 m

27 m



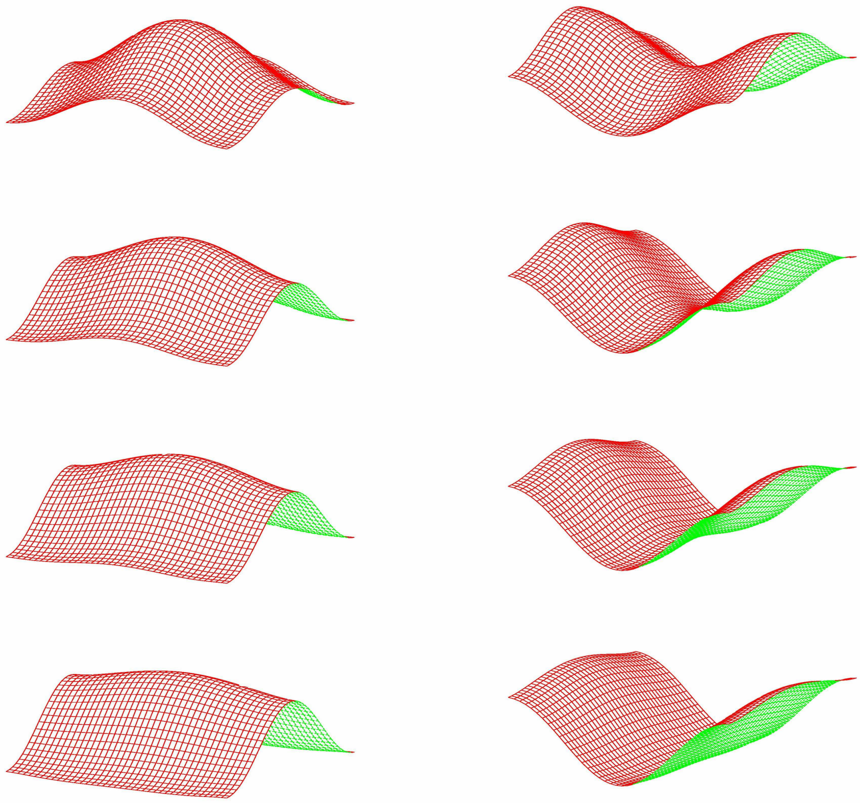
Width

27 m

27.1 m

27.2 m

27.5 m



Some publications

- B. Molin On the piston and sloshing modes in moonpools, *J. Fluid Mech.*, 2001.

- B. Molin, F. Remy, O. Kimmoun, Y. Stassen Experimental study of the wave propagation and decay through a rigid ice-sheet, *Applied Ocean Res.*, 2002.

- B. Molin, F. Remy, A. Camhi, A. Ledoux Experimental and numerical study of the gap resonance in-between two rectangular barges, *Proc. IMAM Conf.*, 2009

- B. Molin On natural modes in moonpools with recesses, *Applied Ocean Res.*, 2017.

- B. Molin, X. Zhang, H. Huang, F. Remy, On natural modes in moonpools and gaps in finite depth, submitted