Physical multi-phase flow model applied to aqueous foam shock tube experiments

J.A. Redford[1], J.-M. Ghidaglia[1], S. Faure[2]

[1]CMLA, ENS Paris-Saclay, CNRS, Université Paris-Saclay, France.
[2]Laboratoire de Mathématiques d'Orsay, Université Paris-Sud XI, France.

16th of October 2017

Outline

Introduction
- Background
- Previous work

Method
- Added mass term
- Experimental and simulation parameters

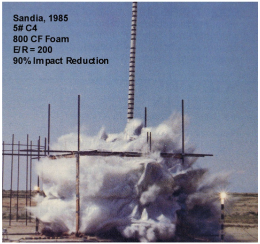
Simulation compared with experiment

Profile data
- Basic quantities
- Kinetic energy budget

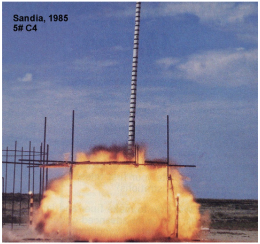
Conclusion

General introduction to SIMATOD[1]

- Suppression of blast waves by aqueous foams
  - Contain hazardous materials
- A challenging multi-phase problem
  - Capture attenuation and sound speed changes
  - Foam contains air, water and possibly some water vapour
  - Quantifying exchanges of mass, momentum & energy between phases is vital when making accurate predictions



with foam



without foam

[1] SIMulation de l'ATténuation des Ondes de Détonation

Introduction : 7 equation model

Model developed by Faure et al. (2011) and D'Alesio et al. (2015)

- 2 phases with possibility of 2 gasses (air and water vapour)
  - Single pressure p
  - 2 velocities u_g, u_w and 2 temperatures T_g, T_l
  - 3 equations of state for $(\rho, e, ...) = f(p, T)$
- Includes drag, heat exchange and phase change source terms
- Extension of 6 equation (2 fluid) approach proposed by Ghidaglia et al. (2001)

Also, Quicksteam developed by Labourdette et al. (2017)

- Equation of state algorithm for water and steam
- Based on IAPWS[2] correlations
  - Details is W. Wagner, H.-J. Kretzschmar (2008)

[2] International Association for the Properties of Water and Steam

Multi-phase flow problems

To solve multi-phase flow problems we must consider

- Exchanges between phases : heat, phase change, drag, added mass,...
- Hence, necessity for two velocities and temperatures to explicitly model exchanges

However, up to now

- Model has not captured observed **reduction in sound speed**

So need to include added mass source terms

- Literature shows this to be important, see Atkinson & Kytömaa (1992)

Added mass source terms

Added mass source terms for momentum can be defined as

$$f_g \equiv -\kappa \frac{\alpha_g \alpha_w \rho_g \rho_w}{\alpha_g \rho_g + \alpha_w \rho_w} \frac{\partial (u_g - u_w)}{\partial t} \text{ and } f_w = -f_g$$

volume fraction α, density ρ, velocity u of gas $_g$ and water $_w$.

- κ is a non-dimensional constant
- Strength of added mass effect (κ) depends on type of multiphase problem
  - For example, difference between water droplets and foam, which has a membrane type structure (as will be seen)
  - The physical processes behind the change in added mass effect will not be tackled in this work

7 equation model with added mass

Including added mass in governing equations gives

$$
\begin{aligned}
\delta_t + \; \dots \; &= -Q_s\,,\\
(\alpha_g \rho_g)_t + \; \dots \; &= Q_s\,,\\
(\alpha_w \rho_w)_t + \; \dots \; &= -Q_s\,,\\
(\alpha_g \rho_g u_g)_t + \; \dots \; &= Q_s u_i + C_{\text{drag}}(u_w - u_g) + f_g\,,\\
(\alpha_w \rho_w u_w)_t + \; \dots \; &= -Q_s u_i + C_{\text{drag}}(u_g - u_w) + f_w\,,\\
(\alpha_g \rho_g E_g)_t + \; \dots \; &= Q_s \left(h_{is} + \frac{|u_i|^2}{2} \right) + C_{\text{drag}}(u_w - u_g) \cdot u_i + Q_{is} + f_g \cdot u_g\,,\\
(\alpha_w \rho_w E_w)_t + \; \dots \; &= -Q_s \left(h_{iw} + \frac{|u_i|^2}{2} \right) + C_{\text{drag}}(u_g - u_w) \cdot u_i + Q_{iw} + f_w \cdot u_w\,,
\end{aligned}
$$

total energy $E = e + \frac{1}{2} u^2$, internal energy e, total enthalpy $H = E + \frac{p}{\rho}$.

Re-derive matrices necessary for numerical solution

- Using Maple software
- Vast increase in number of terms!

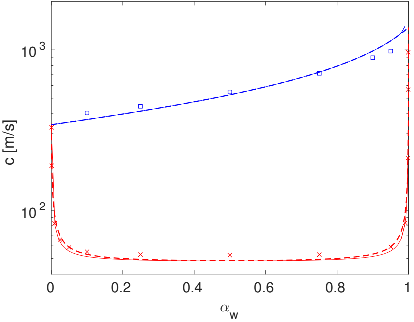
Model sound speed

Compare sound speed calculated from equations (using Maple), simulation and analytical equation for $\kappa = 55$

- In Maple, set $u_g = u_w = 0$ and eigenvalues yield sound speed
- In simulation, propagation speed of small perturbation
- Compare with

$$c^2 = \left[\frac{\alpha_g(\rho+\kappa\rho_g)}{\rho(1+\kappa)\rho_g} + \frac{\alpha_w(\rho+\kappa\rho_w)}{\rho(1+\kappa)\rho_w} \right] \frac{\rho_g \rho_w c_g^2 c_w^2}{\alpha_g \rho_w c_w^2 + \alpha_w \rho_g c_g^2}$$

- Blue, both ideal gasses
- Red, stiffened gas (liquid)/ideal gas
- Pure phase (α_w or $\alpha_g = 0$) velocity maintained



- Large reduction in sound speed for case with liquid

A good comparison!

Numerical solution robustness

Substantial problems encountered even when trying to achieve results at moderate pressure ratios, let alone pressures for explosives (e.g. $\sim 2\,\mathrm{GPa}$).

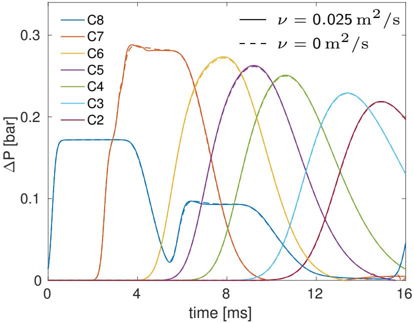
- Focus on simulations at low pressures to confirm the physics

Instability from inadequate modelling of exchange terms

- Improved stability with added mass (stronger coupling between phases)

Develop further strategies to improve code stability

- Add diffusion; viscosity/thermal
- Tests show only a small effect on results

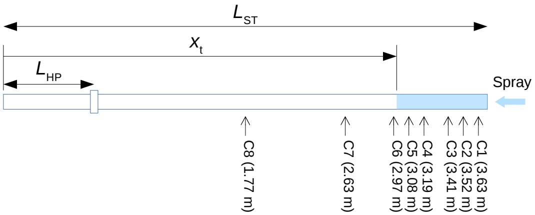


Shock tube laboratory experiments

Two experimental datasets from the literature, where test section contains

- Droplets from atomizing spray (Jourdan et al., 2010)
  - In vertical orientation
- Foam (Jourdan et al., 2015)

with pressure measurements from 8 locations (see diagram)



Experimental parameters

Range of Mach numbers

- Droplets $M = 1.1$ & 1.5; Foam $M = 1.07, 1.3$ & 1.5

Two foam expansion ratios

- $\phi = 30$ & 80, note volume fraction $\alpha = 1/\phi$
- Sound speeds of 50 & 70 m/s, respectively

Left boundary condition

- Wall replicates air in high pressure (HP) chamber
- Neumann replicates sulphur hexafluoride (SF_6)

No.	M	ϕ	P_{HP} [bar]	ΔT_{HP} [K]	BC_L [m]	x_t	$\alpha_w \times 10^6$ $x < x_t$	$\alpha_w \times 10^2$ $x_t < x$	N_x
1	1.1	77	1.85	0	W	3.12	1.3	1.3	300
2	1.5	77	5.7	60	W	3.12	1.3	1.3	100
3	1.5	100	5.7	65	W	3.12	1	1	100
4	1.07	30	1.38	0	W	2.75	3.3	3.3	300
5	1.3	30	3.8	5	N	2.75	1	3.3	300
6	1.5	80	6.5	55	W	2.75	1.25	1.25	300
7	1.5	30	6.5	70	W	2.75	1	3.3	300
8	1.3	80	3.4	15	N	2.75	3.3	1.25	300
9	1.3	30	3.4	5	N	2.75	1	3.3	300

List of cases run with parameters and reference to experiment for comparison. Cases 1 to 3 have $\kappa = 0$ and all others have $\kappa = 55$. BC_L is the left boundary condition, which is either wall, denoted by W, or Neumann, denoted by N.

Simulation parameters

Increasing resolution not always beneficial

- Optimum resolution is 300 cells
- Lower resolutions used for $M = 1.5$ spray cases that are more unstable

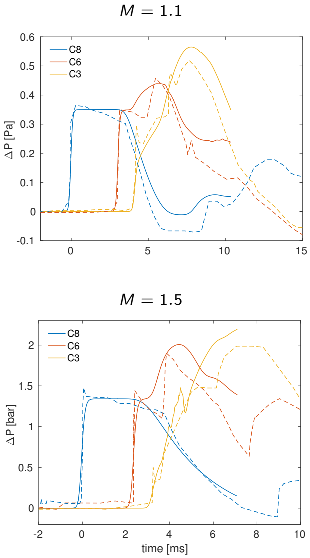
Initially very little vapour and liquid temperatures remain well below saturation

- No evaporation (phase change), hence vapour phase not important
- Use air equation of state for vapour to improve stability

Shock tube with droplets

Water droplets from atomizing spray in test section

- No sound speed change from air, use $\kappa = 0$
- Include drag term with 500μm diameter droplets
- Difficulties with code stability
  - Reduce resolution to 100 cells for $M = 1.5$
  - Simulations tend to fail when pressure wave hits test section wall
- Good comparison between simulation and experiment

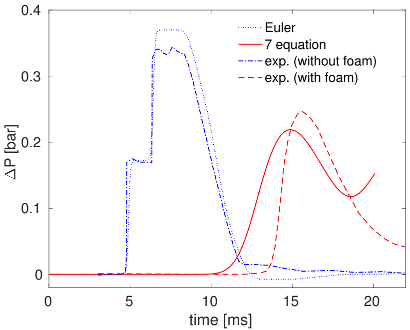


Foam at $M = 1.07$

Foam in test section

- Experiment without foam compared to Euler model (single fluid)
- $\phi = 30$ so we take $\kappa = 55$
- C2 close to test section wall
- Demonstrates large time delay
- Substantial reduction in peak pressure
- Excellent comparison between simulation and experiment

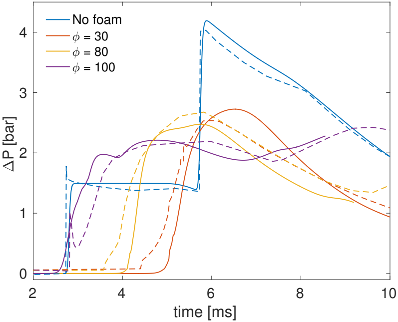
$M = 1.07$ & $\phi = 30$ at C2



Foam and droplets at $M = 1.5$

Including Foam $\phi = 30, 80$ and droplet $\phi = 100$

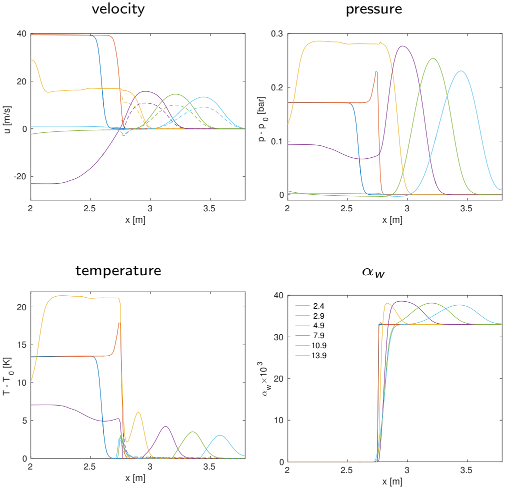
- M $= 1.5$ droplets, problems with calculation stability
  - Necessary to reduce resolution to 100 cells
- $\phi = 100$ is droplet case, hence $\kappa = 0$ is used
  - Slight delay in experiment suggests $\phi = 100$ may have small sound speed reduction
- Excellent correspondence over wide range of parameters



$M = 1.5$ at C4; — Experiment, - - - Simulation

$M = 1.07$ & $\phi = 30$ profiles

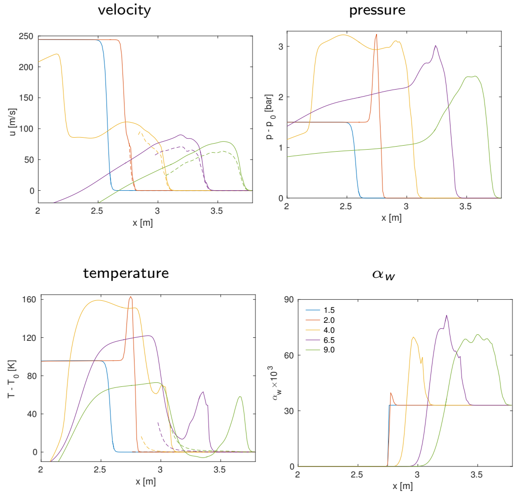
- Yields complete set of statistics
- Demonstrates internal processes
- Large velocity reduction & pressure increase
- Large temperature drop
  - Cooling effect of the liquid
- Volume fraction shows greater concentration of liquid



Foam starts at $x = 2.75$ m;
— Gas, - - - Liquid

$M = 1.5$ & $\phi = 30$ profiles

- Similar to low Mach number
- Most noticeable difference in volume fractions
  - Liquid pushed further right and becomes more concentrated
- Liquid temperatures remain low
  - No phase change



Foam starts at $x = 2.75$ m;
— Gas, - - - Liquid

$M = 1.5$ & $\phi = 30$ kinetic energy budget

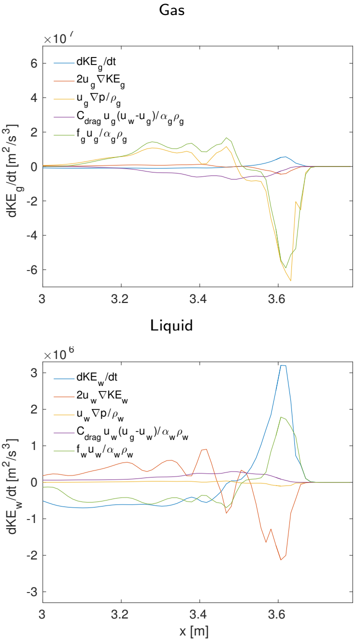
Kinetic energy budget equations:

$$\frac{\partial k_g}{\partial t} + 2u_g \nabla k_g + \frac{u_g}{\rho_g} \nabla p = C_{\text{drag}} \frac{u_g(u_w - u_g)}{\alpha_g \rho_g} + f_g \frac{u_g}{\alpha_g \rho_g}$$

$$\frac{\partial k_w}{\partial t} + 2u_w \nabla k_w + \frac{u_w}{\rho_w} \nabla p = C_{\text{drag}} \frac{u_w(u_g - u_w)}{\alpha_w \rho_w} + f_w \frac{u_w}{\alpha_w \rho_w}$$

Relative importance of terms & transfers between liquid and gas phases

- Gas; pressure gradient & added mass dominate
- Liquid; acceleration, advection and added mass dominate dominate
- Small contribution from drag



Conclusions

- Including added mass term captures sound speed reduction due to multi-phase flow
- Re-derived 7 equation model with added mass and implemented in 1D (also radial & spherical coordinates) code
- Good comparison with shock tube experiments over a wide range of parameters
  - Including liquid droplets and foams in test section
- Simulations yield extensive profile data that highlights internal processes and exchanges between liquid and gas phases
  - Added mass is shown to dominate over drag
- In future, make higher pressures and Mach numbers possible

Thank you!