The Numerical Simulation of Free Surface Instabilities using an Immersed Interface Method for Surface Tension

Ronald Remmerswaal   Arthur Veldman

University of Groningen

Tuesday October 17th, 2017

Introduction

Variability of impact pressures

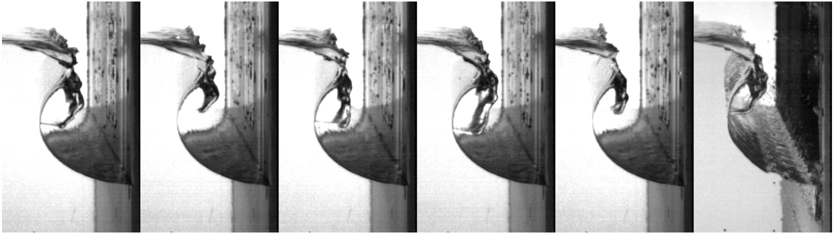

Figure: Results from the Sloshel project (2009) @ scale $1:6$.

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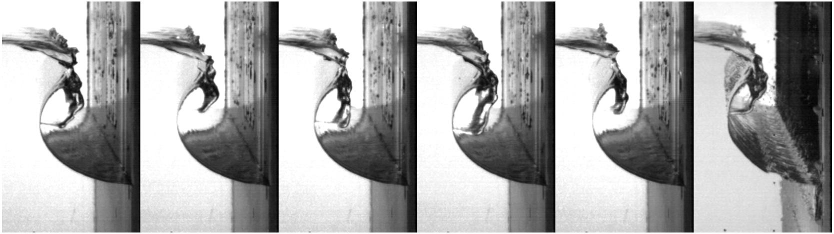

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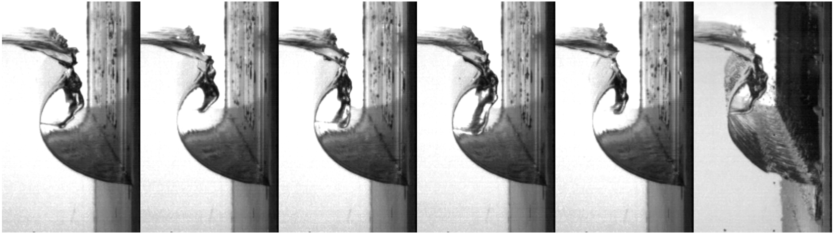

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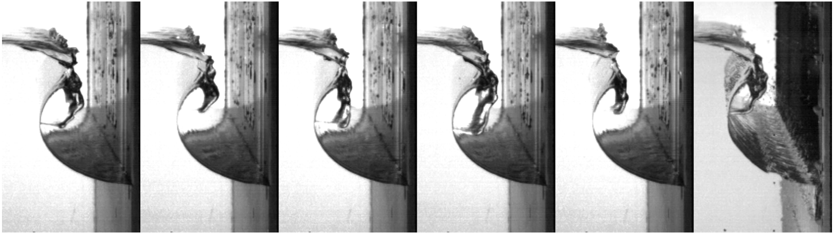

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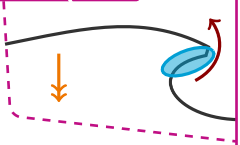
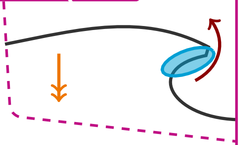
Final goal...

...is to capture this variability in a numerical model



- Plateau-Rayleigh: break-up of liquid filaments

[1] Chandrasekhar, Hydrodynamic and Hydromagnetic Stability (1961)



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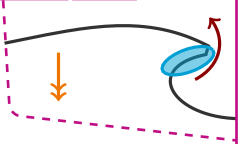

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  - Inviscid linear stability analysis: unstable if[1]

$$\underbrace{\frac{1}{1+\mathcal{R}_\rho}}_{\approx 1}\underbrace{\left(\frac{\text{inertia}}{\text{surface tension}}\right)}_{\text{Weber number}} + \underbrace{\left(\frac{\text{gravity}}{\text{surface tension}}\right)}_{\text{Bond number}} > 1$$

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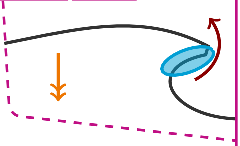

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  - Most unstable for perturbations of wavelength λ^*_{KH}

$$\lambda^*_{\text{KH}}=\frac{4\pi(1+\mathcal{R}_\rho)}{3}\frac{\sigma}{\rho^g U^2}\approx\underbrace{\frac{0.0335}{U^2}}_{LNG+NG}$$

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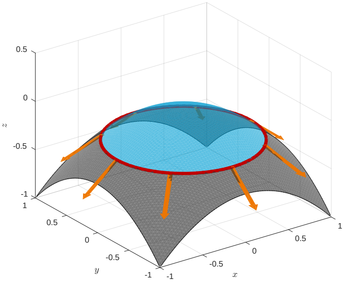

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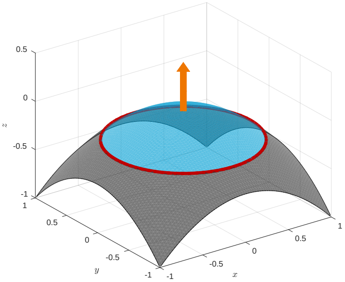

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$$0 = \lim_{|P|\to 0} \int_P \left([\![p]\!] + \sigma\kappa \right) \mathbf{n} \; dS$$

yields Laplace's equation

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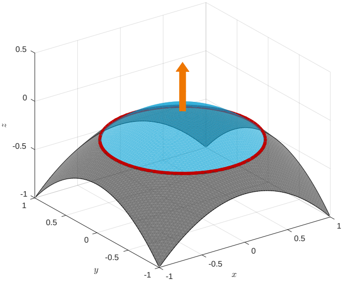
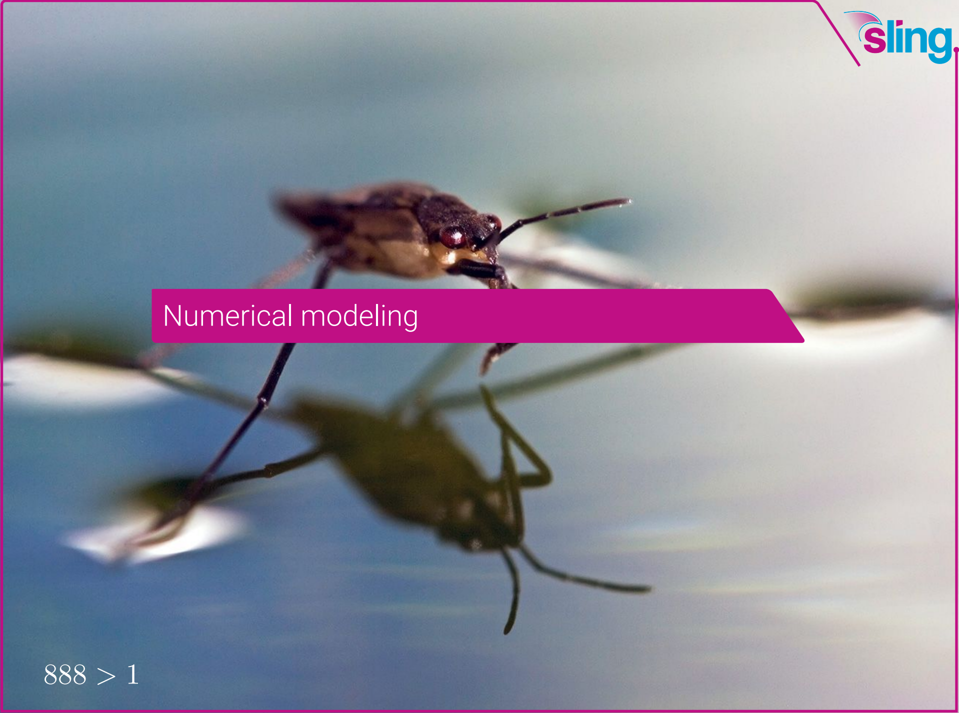
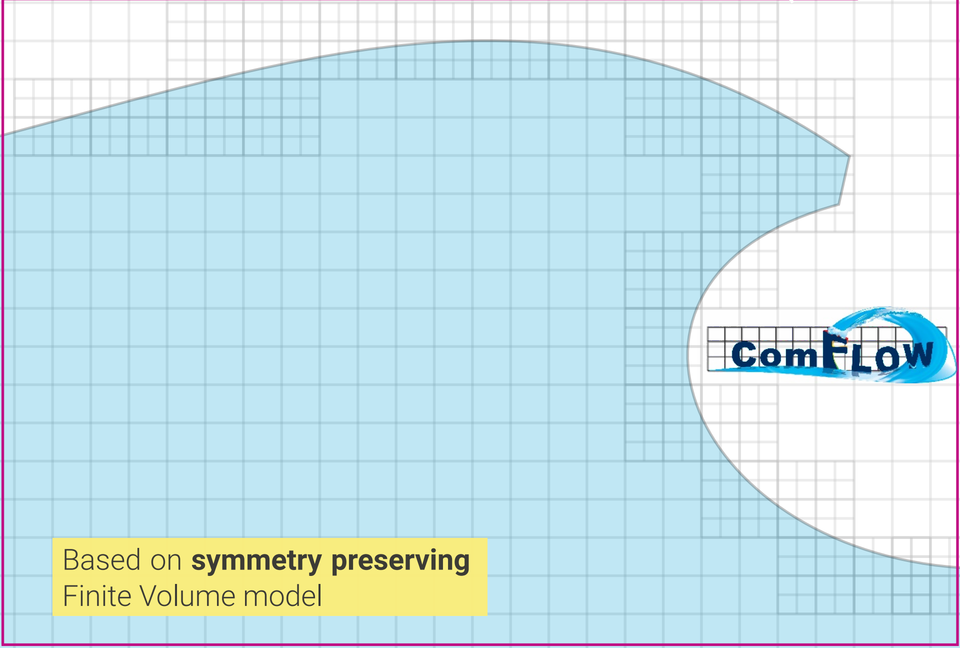




Numerical modeling

888 > 1

The underlying Navier-Stokes model



Based on **symmetry preserving** Finite Volume model

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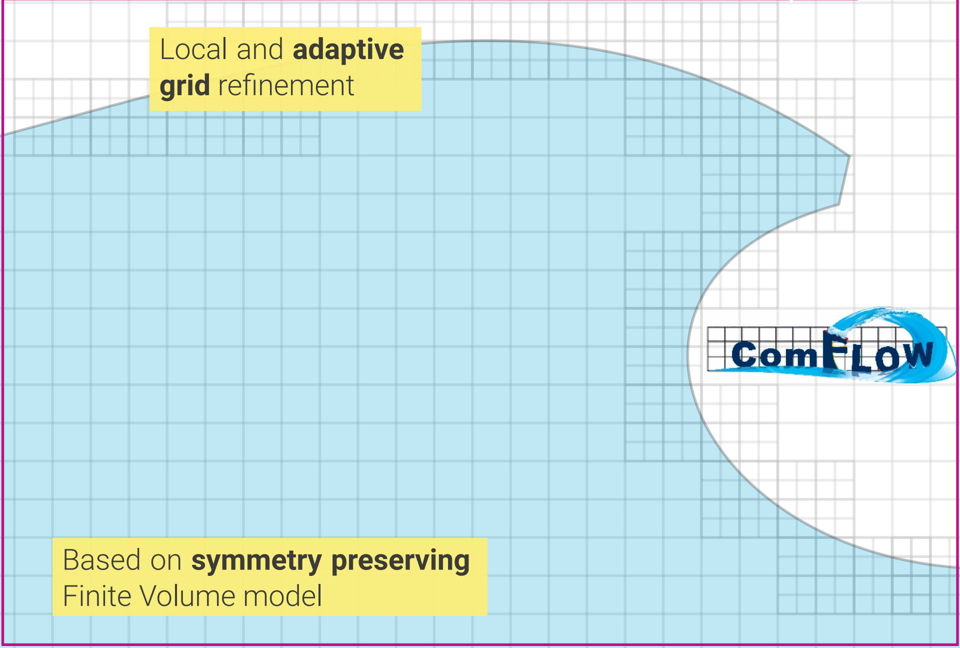

Local and **adaptive grid** refinement

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OpenMP and **MPI**

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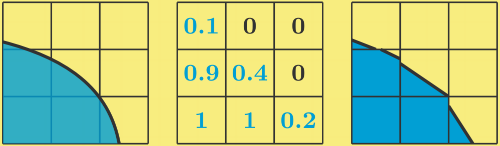
Sharp interface using PLIC



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Desired properties

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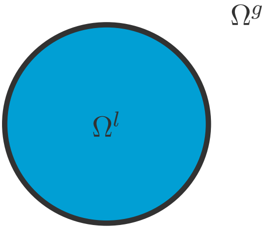

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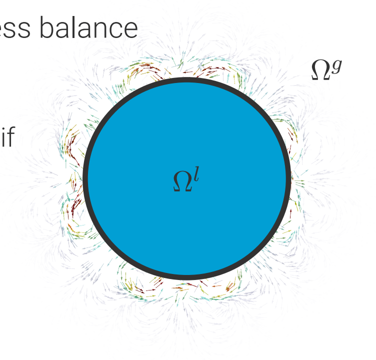

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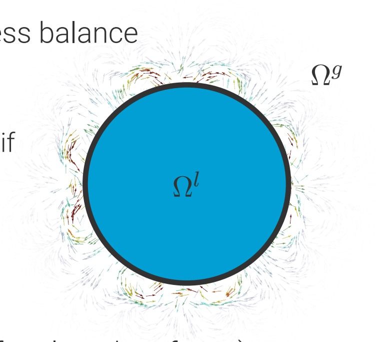

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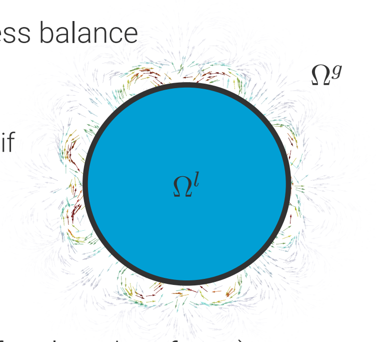

- Conservation of momentum (for closed surfaces)
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- Conservation of total energy

$$E_{\text{kin}}(t) + \sigma |I(t)|$$

Continuum Surface Force

Approximating the surface force with a volume force [2]

$$\int_P \sigma \kappa \mathbf{n} \; dS = \lim_{\epsilon \to 0} \int_\Omega \mathbf{S}^\epsilon \; dV$$

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- CSF yields a class of models based on two ingredients:
  - Approximation of curvature $\bar{\kappa}^\epsilon$ (we use a local height function)
  - Discretization of $\nabla\tilde{\chi}^\epsilon$

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Imposing the stress balance (inspired by our one-phase model)

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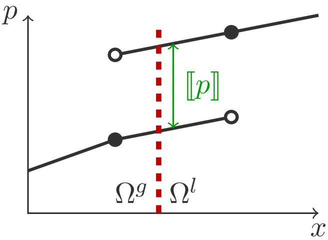

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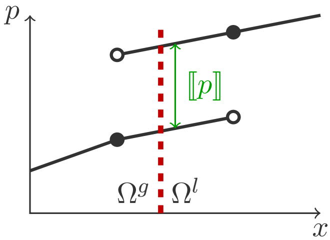

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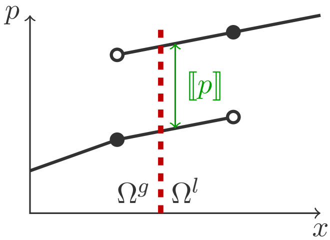

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- Part of a general IIM framework [a]: ‘Two Sided GFM’
- Results in a **sharp** pressure jump

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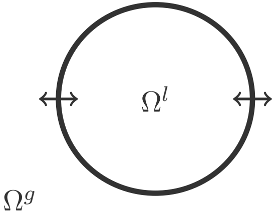
Numerical results

Simulation of a 'floating' 2D droplet of radius R in the absence of gravity, desired steady state:

$$\mathbf{u} = \mathbf{0}, \quad p = \begin{cases} p^{\text{atm}} + \frac{\sigma}{R} & \mathbf{x} \in \Omega^l \\ p^{\text{atm}} & \mathbf{x} \in \Omega^g \end{cases}$$



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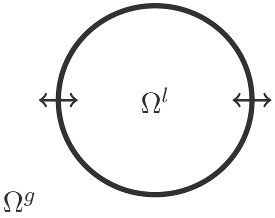

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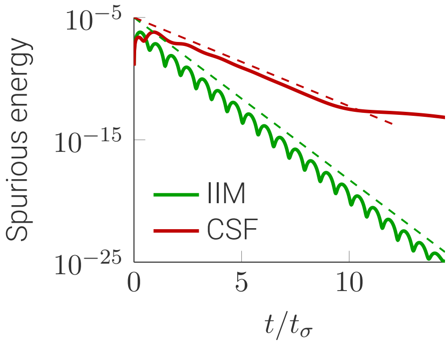

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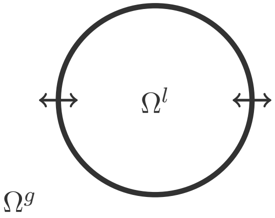

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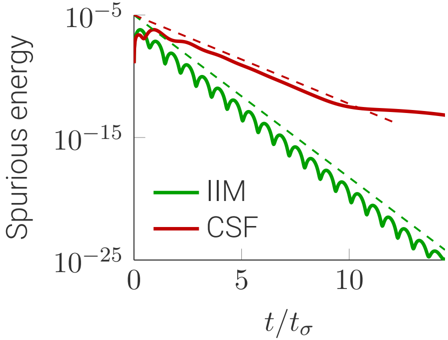

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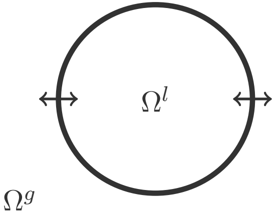

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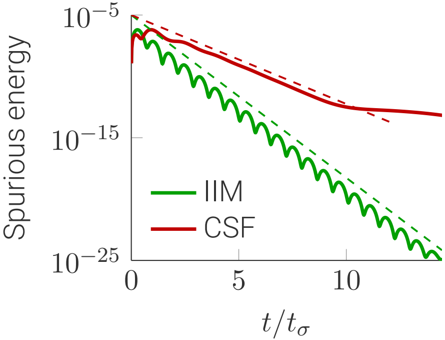

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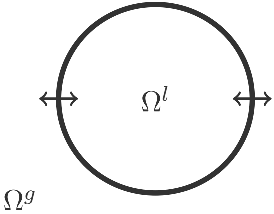

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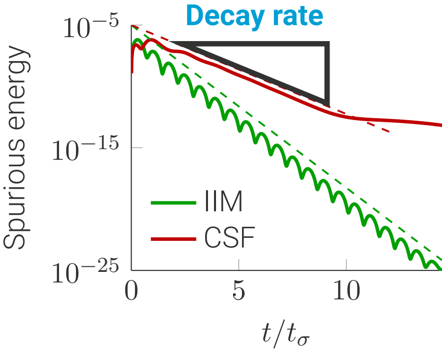

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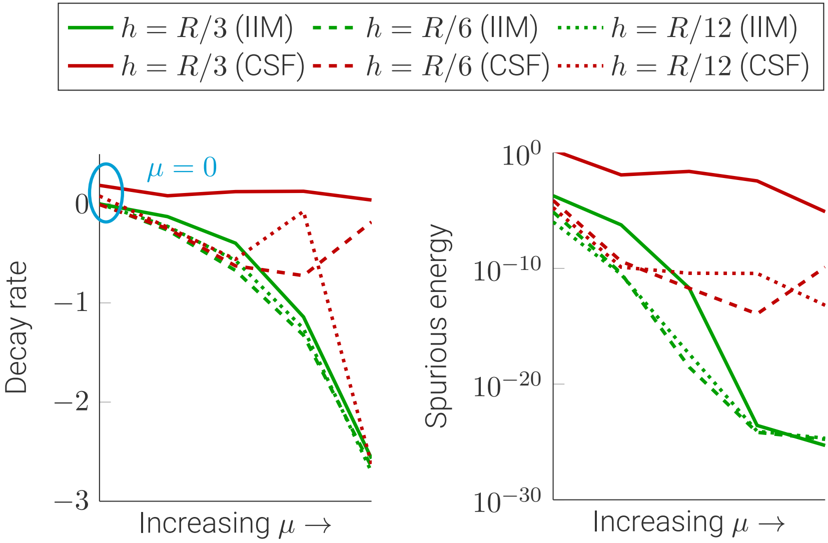

Figure: Spurious energy decay rate

Figure: Spurious energy at $t = t^{\text{end}}$

Preliminary application: effect of **capillarity** on development of a 2D Kelvin-Helmholtz instability

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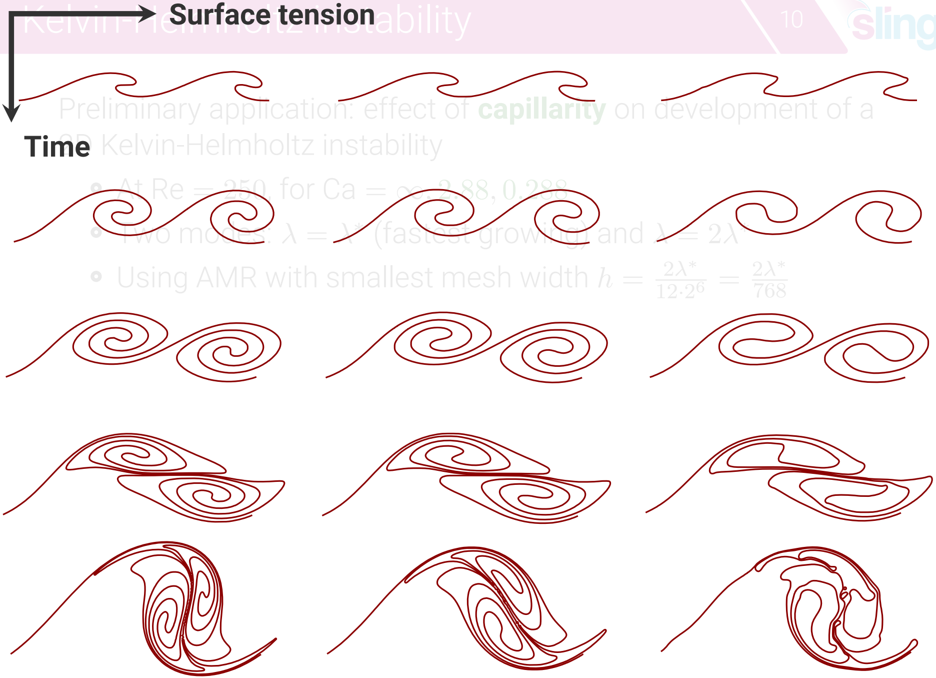

Concluding remarks

Summary and future work

Summary

- *Many* consistent modeling approaches exist
- CSF: Careful choice of ingredients is required!
- IIM, as opposed to CSF, suppresses spurious velocities even at low resolution

Future work

- Simulation and analysis of capillary effects in 3D Kelvin-Helmholtz instabilities
- Exploration of desired properties (conservation of momentum, energy)
- Other applications of IIM framework in two-phase modeling: e.g. density, contact discontinuities

That’s all

Questions?