
冨嶽三十六景
神奈川沖
浪裏

STW Perspectief

sling.

Collapse of a (non-)axisymmetric air cavity in water

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Anaïs Gauthier • Detlef Lohse

PHYSICS OF FLUIDS

UNIVERSITY OF TWENTE.

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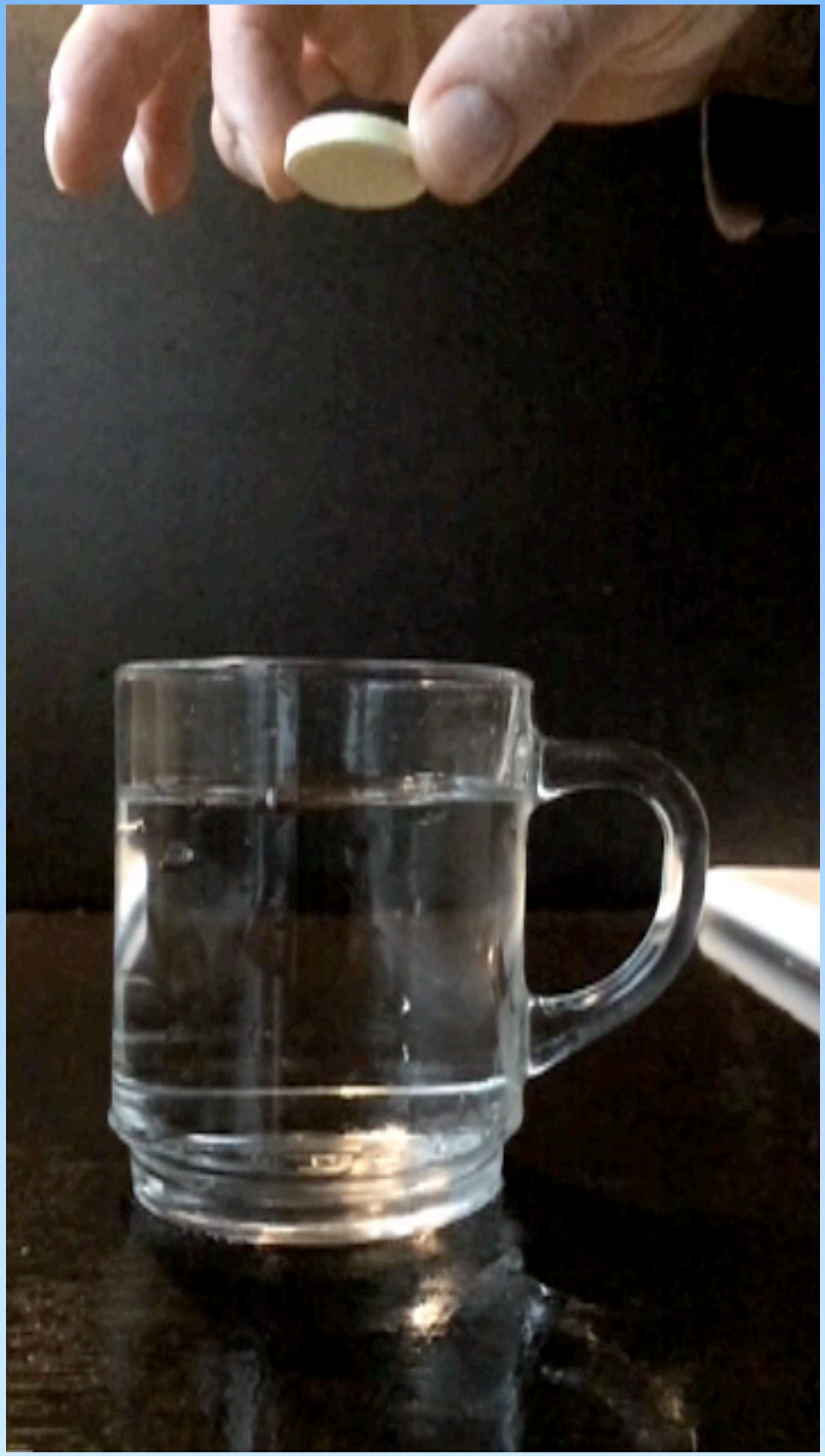
October 17, 2017 – Paris

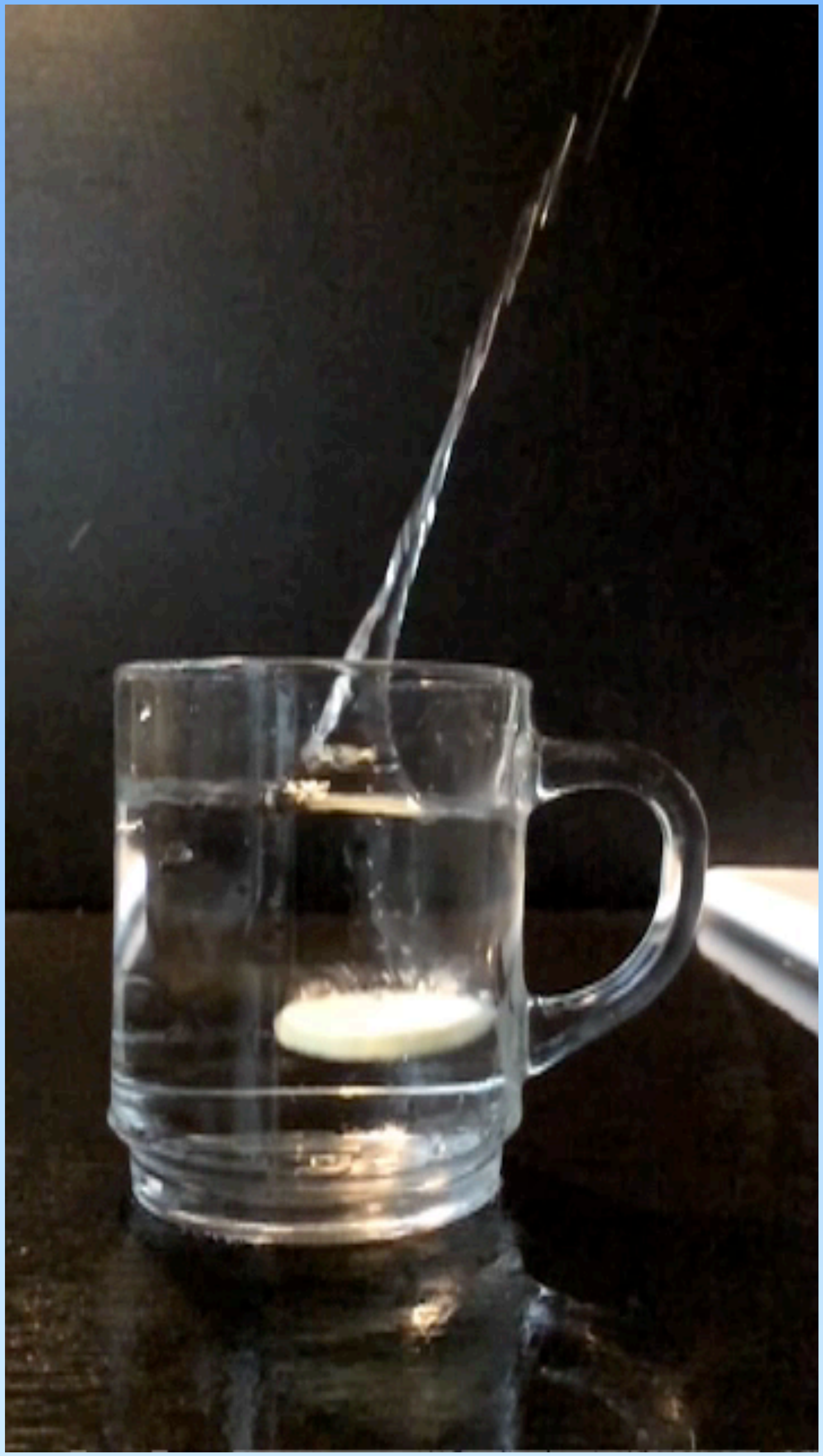
Disclaimer

- **singularity**
- **(controlled) instabilities**
- **air entrapment**
- **experiments, theory & numerics**

Talk by Utkarsh Jain - Wednesday at 16:30

Try this at home



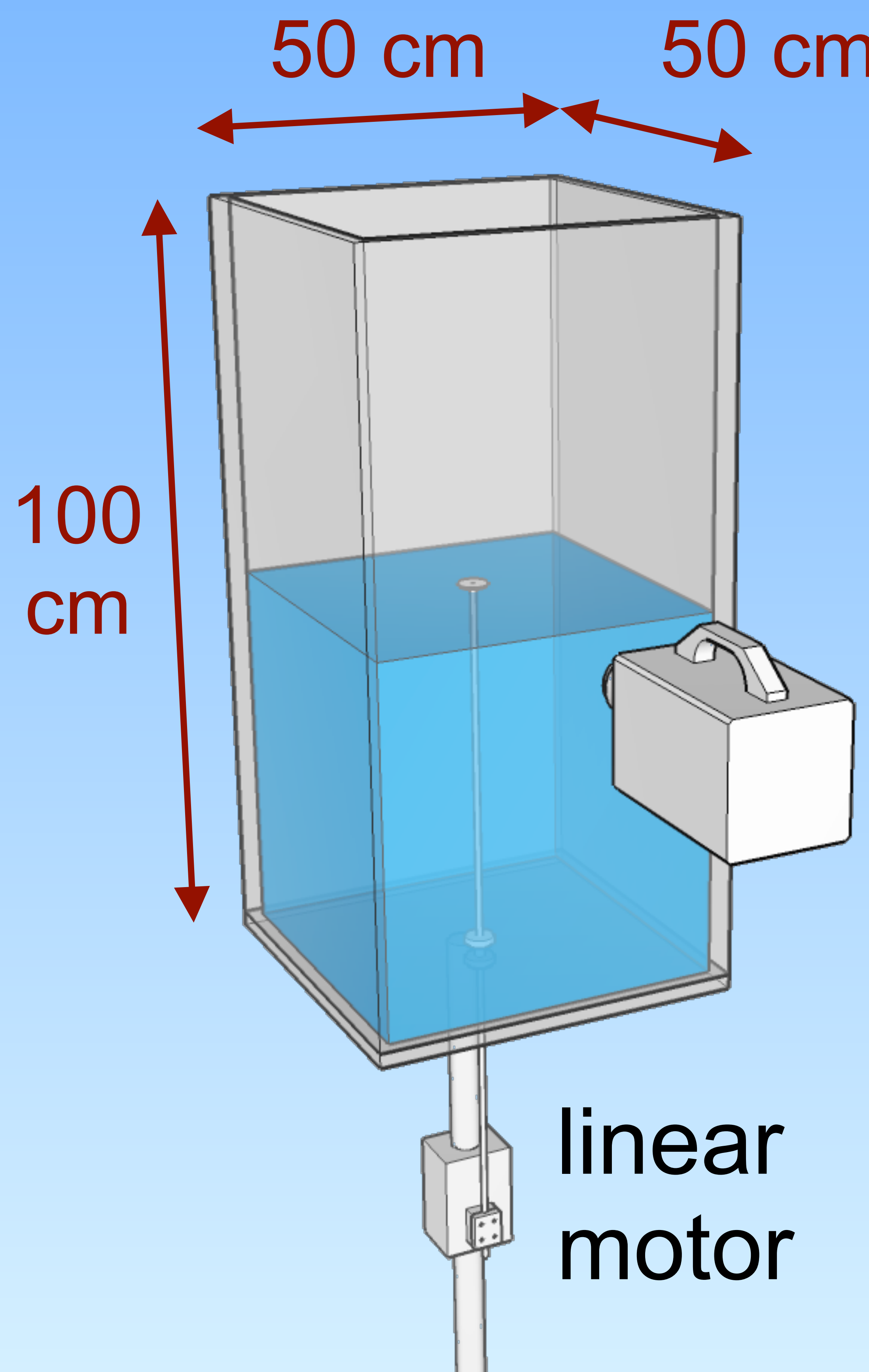


... in our lab


... in our lab


... in our lab

Experimental setup



Disk pulled through interface

V_{impact} = 1.0 m/s

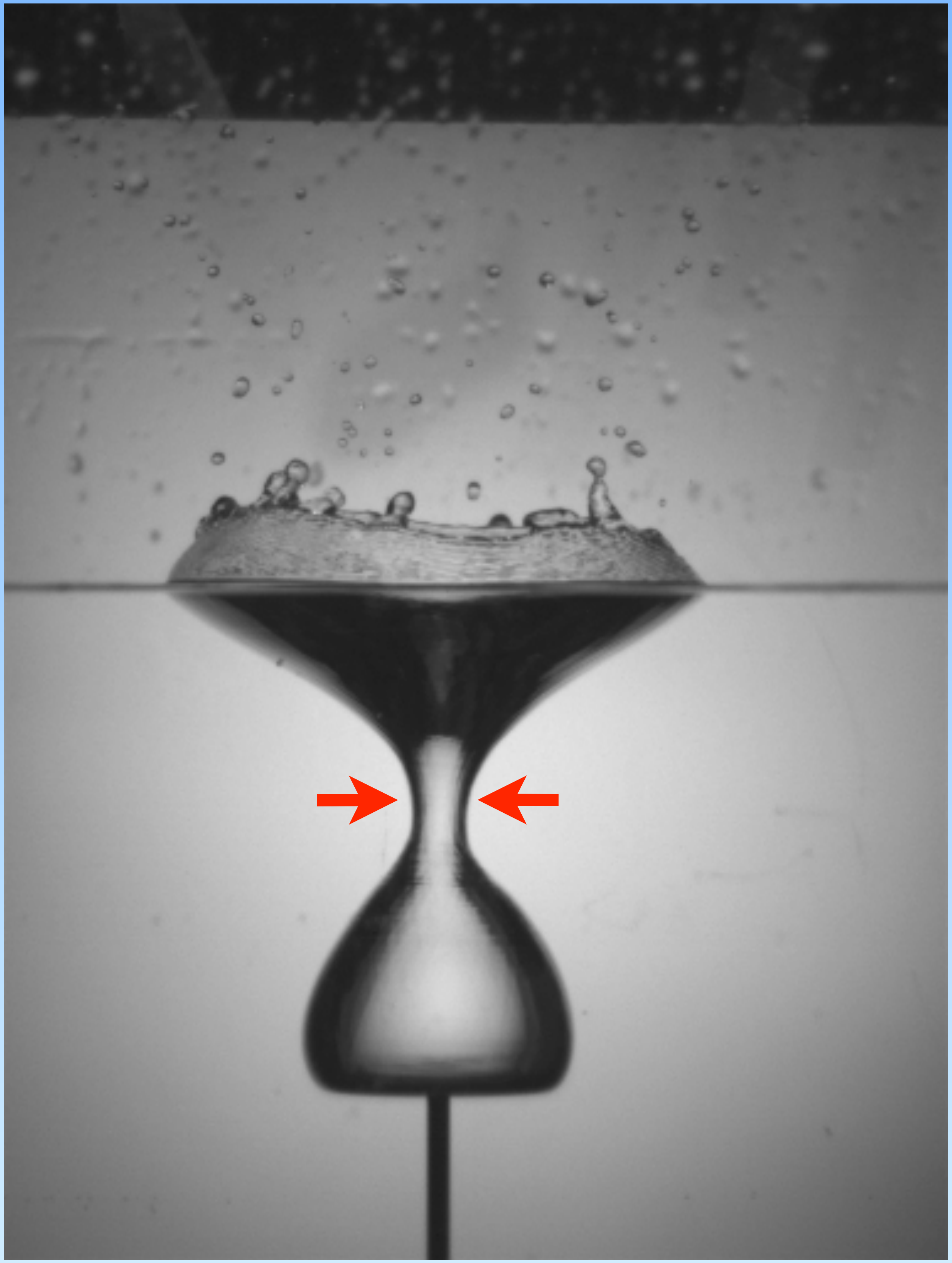
R_{disk} = 0.03 m

camera @ 1000 fps

Series of events

void creation

Series of events



void creation

void collapse

Series of events

void creation

void collapse

jet creation at singularity

Series of events

void creation

void collapse

jet creation at singularity

air-entrainment
"giant bubble"

Dimensional Analysis

Relevant parameters:

- disk radius $R_0 \approx 2$ cm
- disk velocity $V \approx 1$ m/s
- gravity $g = 9.8$ m/s^2
- density $\rho = 1000$ kg/m^3
- viscosity $\eta = 1.0$ mPas
- surface tension σ = 0.074 N/m

$$\mathrm{Fr} = \frac{V^2}{gR_0} \approx 5$$

(Froude number)

inertia and gravity dominant

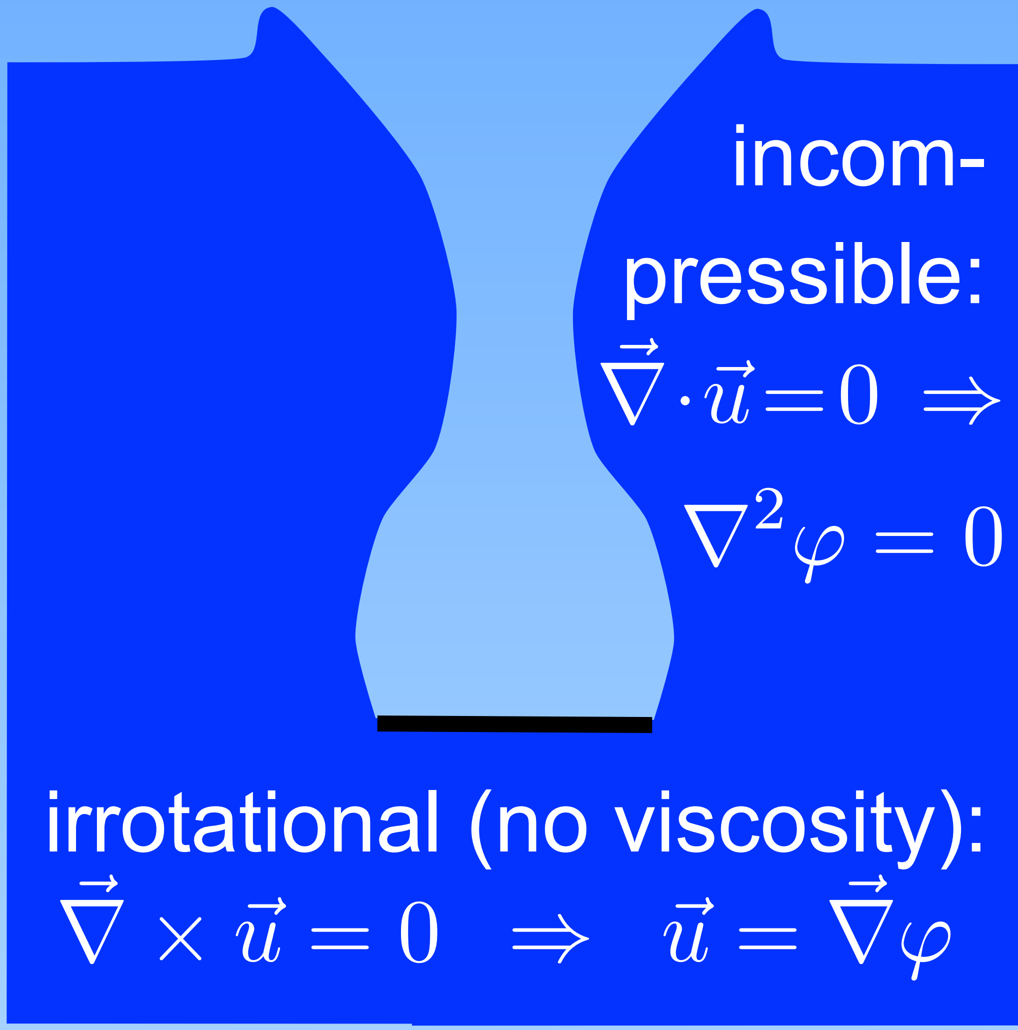
$$\mathrm{We} = \frac{\rho V^2 R_0}{\sigma} \approx 300$$

(Weber number)

$$\mathrm{Re} = \frac{\rho V R_0}{\eta} \approx 20{,}000$$

(Reynolds number)

viscosity unimportant
⇒ potential flow



Boundary Integral simulations

Laplace equation for potential:

$$\nabla^2\varphi = 0$$

is solved as a boundary integral:

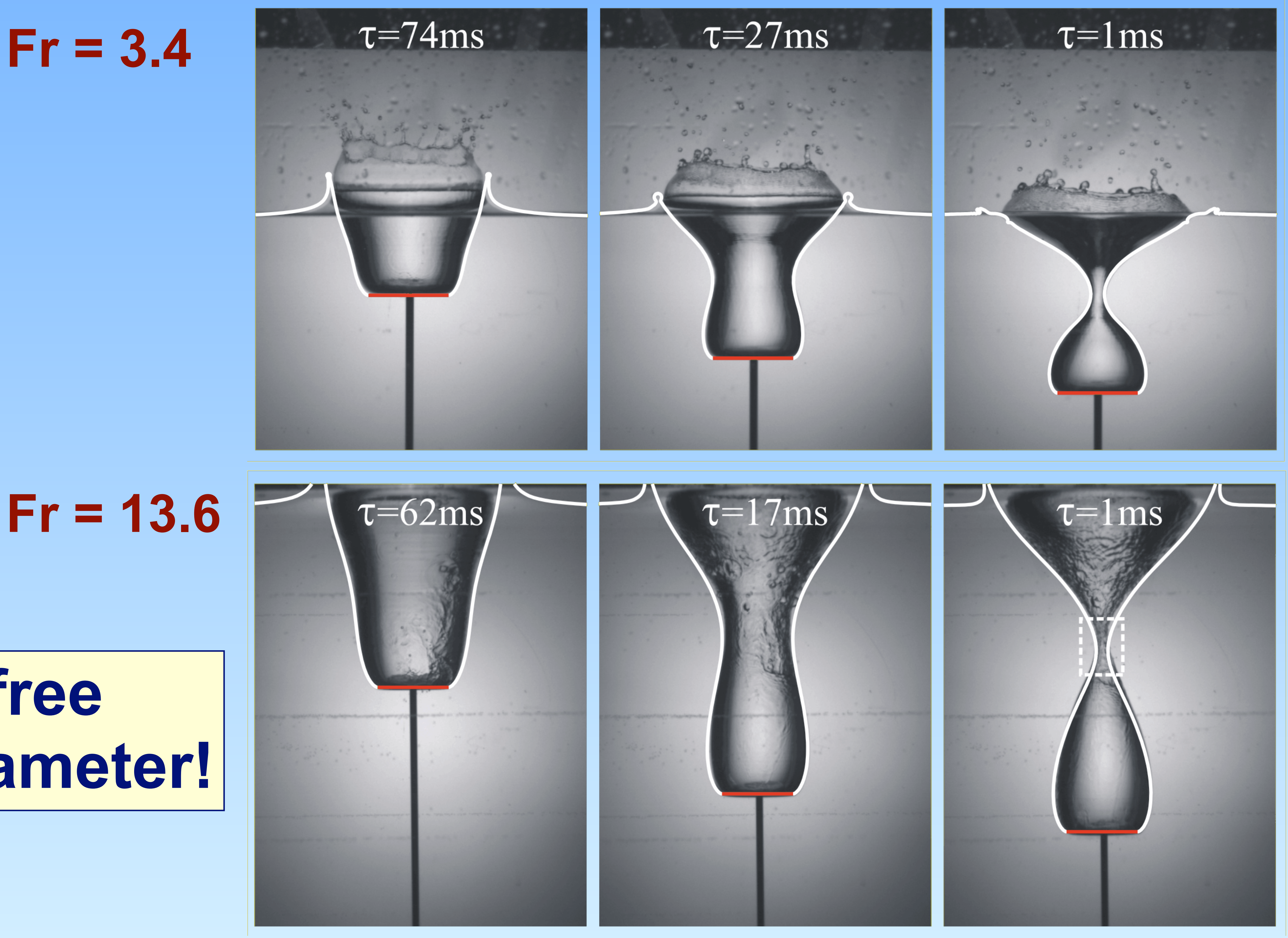
$$\varphi(\vec{x}_0, t) = \iint_{\partial V} \left[G(|\vec{x} - \vec{x}_0|)\vec{\nabla}\varphi(\vec{x}, t) - \varphi(\vec{x}, t)\vec{\nabla}G(|\vec{x} - \vec{x}_0|) \right] \cdot d\vec{A}$$

(Green's third identity)

Unsteady Bernoulli equation provides time evolution:

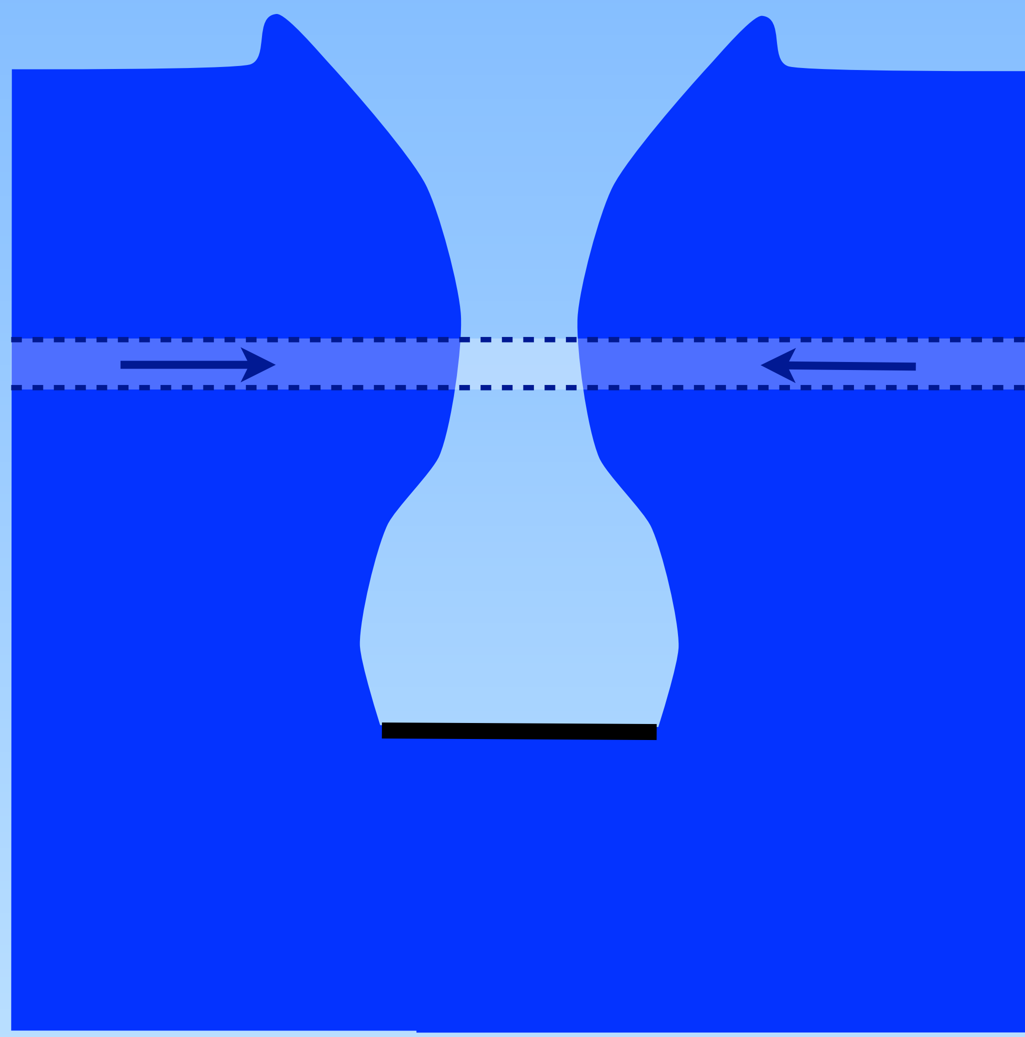
$$\frac{\partial\varphi}{\partial t} + \tfrac{1}{2}|\vec{\nabla}\varphi|^2 = -gz - \frac{\sigma}{\rho}\kappa$$

BI simulation vs. experiments



No free parameter!

Model: Slender cavity limit



Flow in horizontal layers:

→ **Assume potential flow**

→ **Assume axisymmetry**

→ **Neglect axial flow**

needed: equation for
2D fluid flow in layers

2D Rayleigh-Besant equation

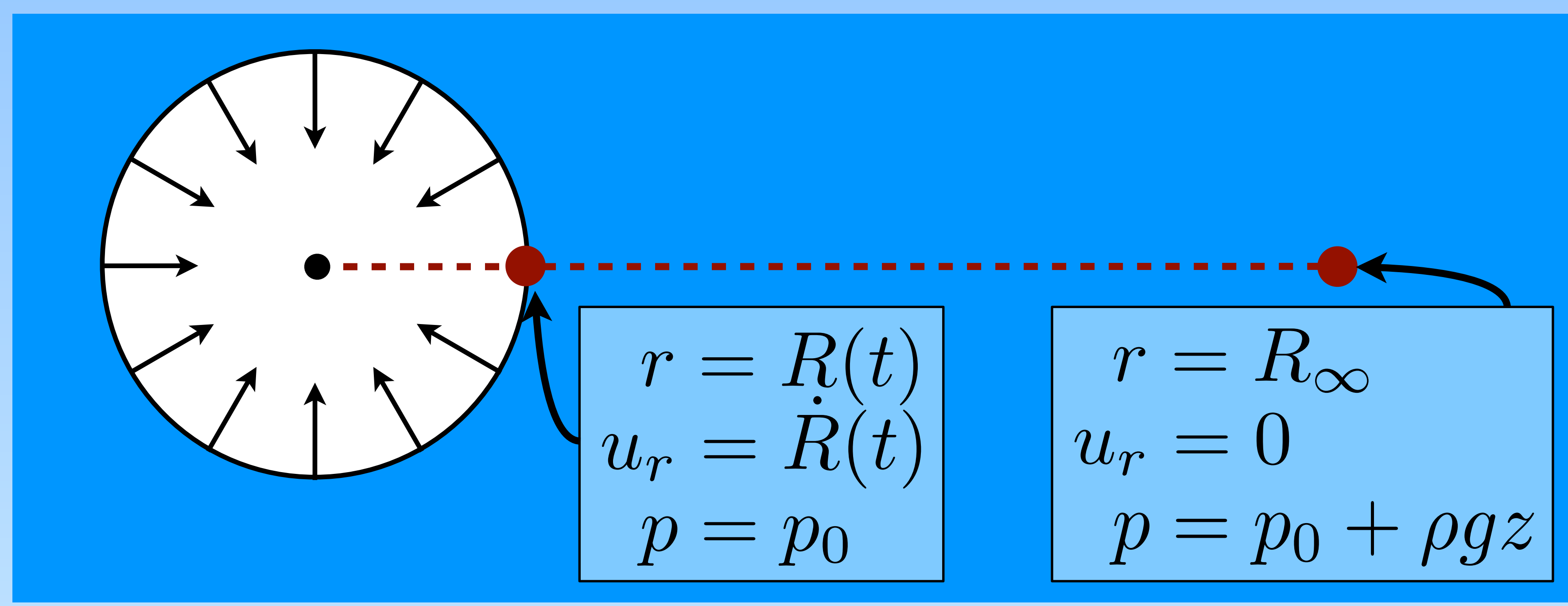
Euler equation in cylindrical coordinates

$$\frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} = -\frac{1}{\rho}\frac{\partial p}{\partial r}$$

Continuity equation

$$r\, u_r = R\,\dot{R}$$

Integrate with boundary conditions:



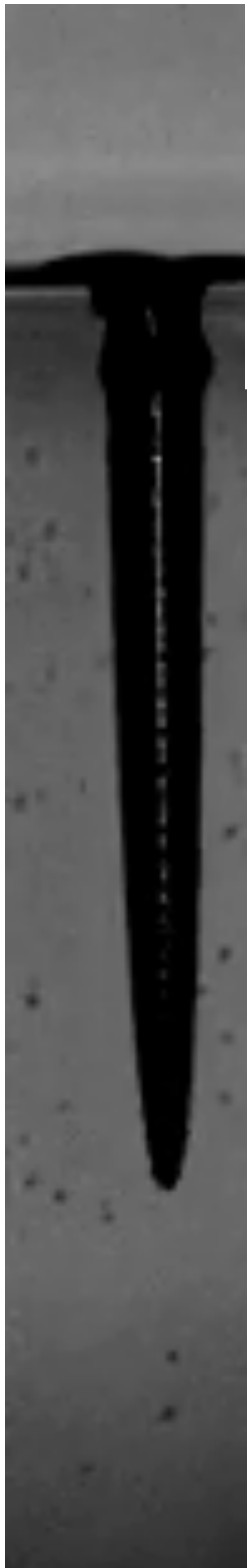
result

$$\frac{d}{dt}(R\dot{R})\,\log\frac{R}{R_\infty} + \tfrac{1}{2}\dot{R}^2 = gz$$

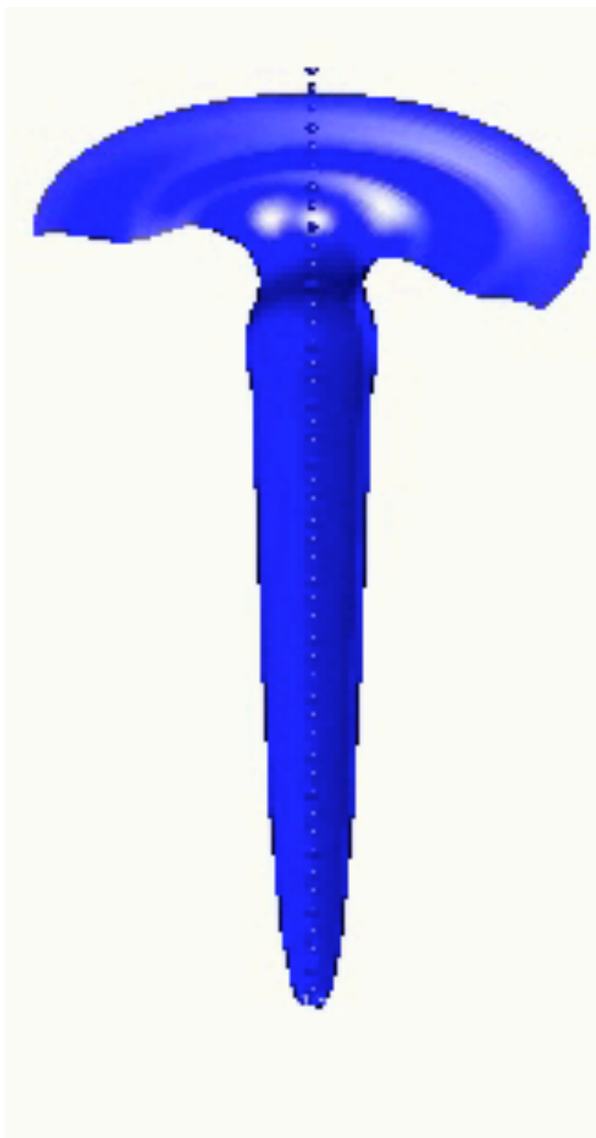
2D Rayleigh equation

R. Bergmann *et al*, PRL **96**, 154505 (2006).

Void creation (at microscale)



Impact of a train of micro droplets (d = 100 µm; V = 12.6 m/s)



BI simulation

$$\frac{d}{dt}(R\dot{R})\,\underbrace{\log\frac{R}{R_\infty}}_{\text{large}} + \frac{1}{2}\dot{R}^2 = \cancel{gz}$$

gravity negligible

$$\frac{d}{dt}(R\dot{R}) = \frac{d^2}{dt^2}\left(\tfrac{1}{2}R^2\right) = 0$$

$$\Rightarrow \quad R(t) = \sqrt{2R_0\dot{R}_0(t+t_0)}$$

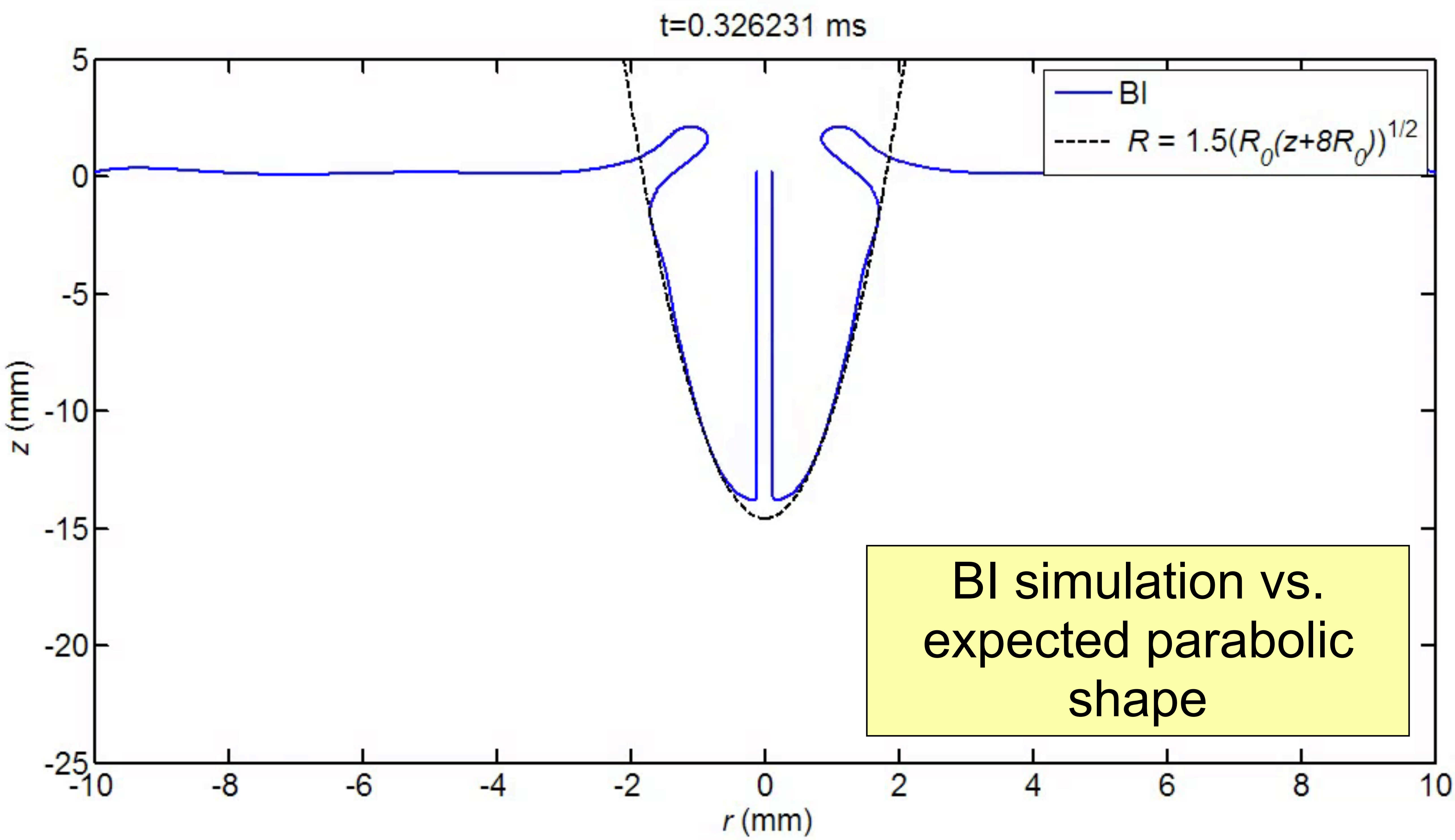
Void created with constant velocity V:

$$t = \frac{z}{V}$$

$$\boxed{R(z) = \sqrt{2R_0\frac{\dot{R}_0}{V}(z+z_0)}}$$

Parabolic shape !

Cavity shape (at microscale)



W. Bouwhuis *et al*, preprint (2014).

Cavity shape (disc impact)

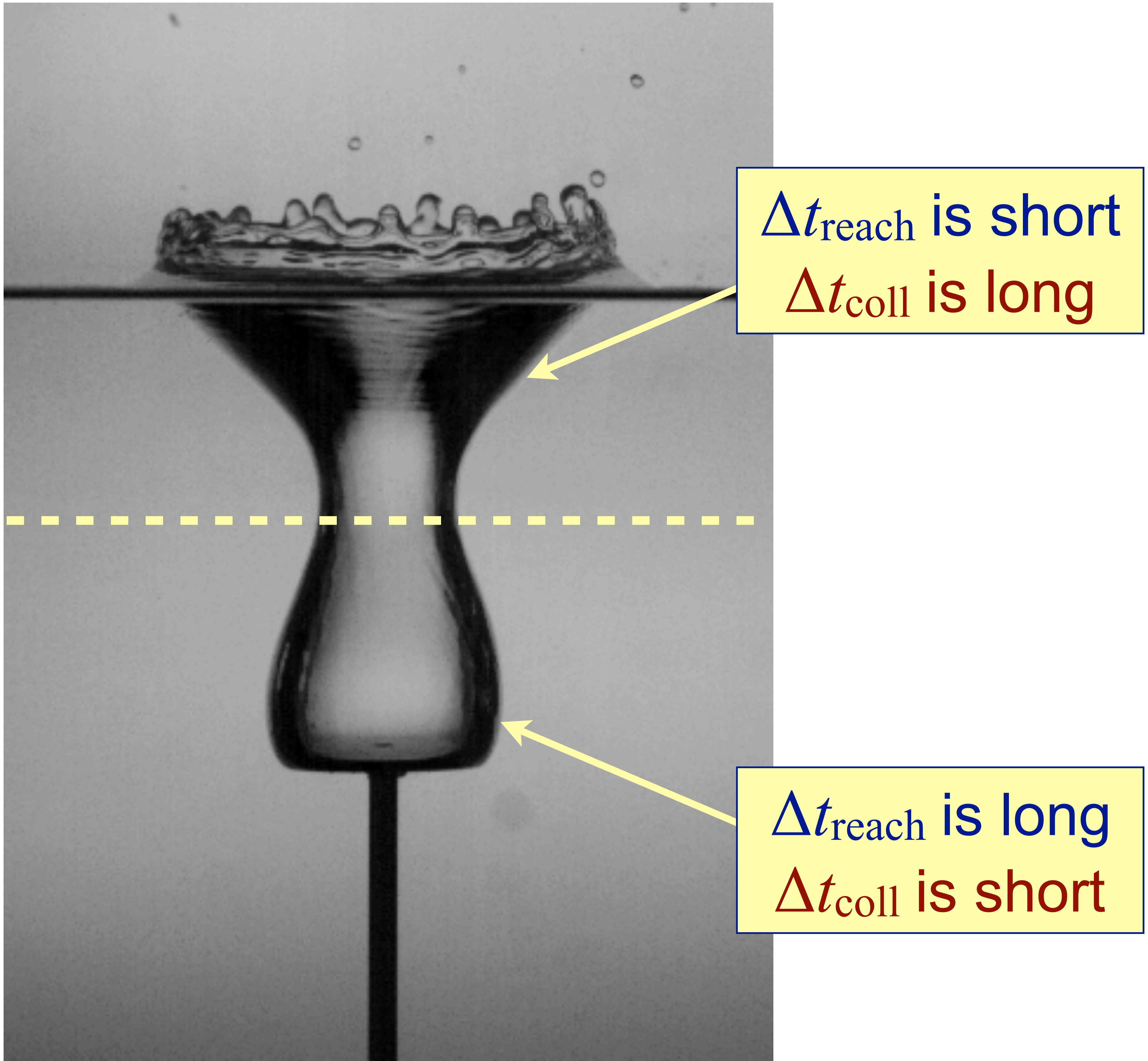
$$\Delta t_{reach} = \frac{z}{V}$$

determined by impact speed

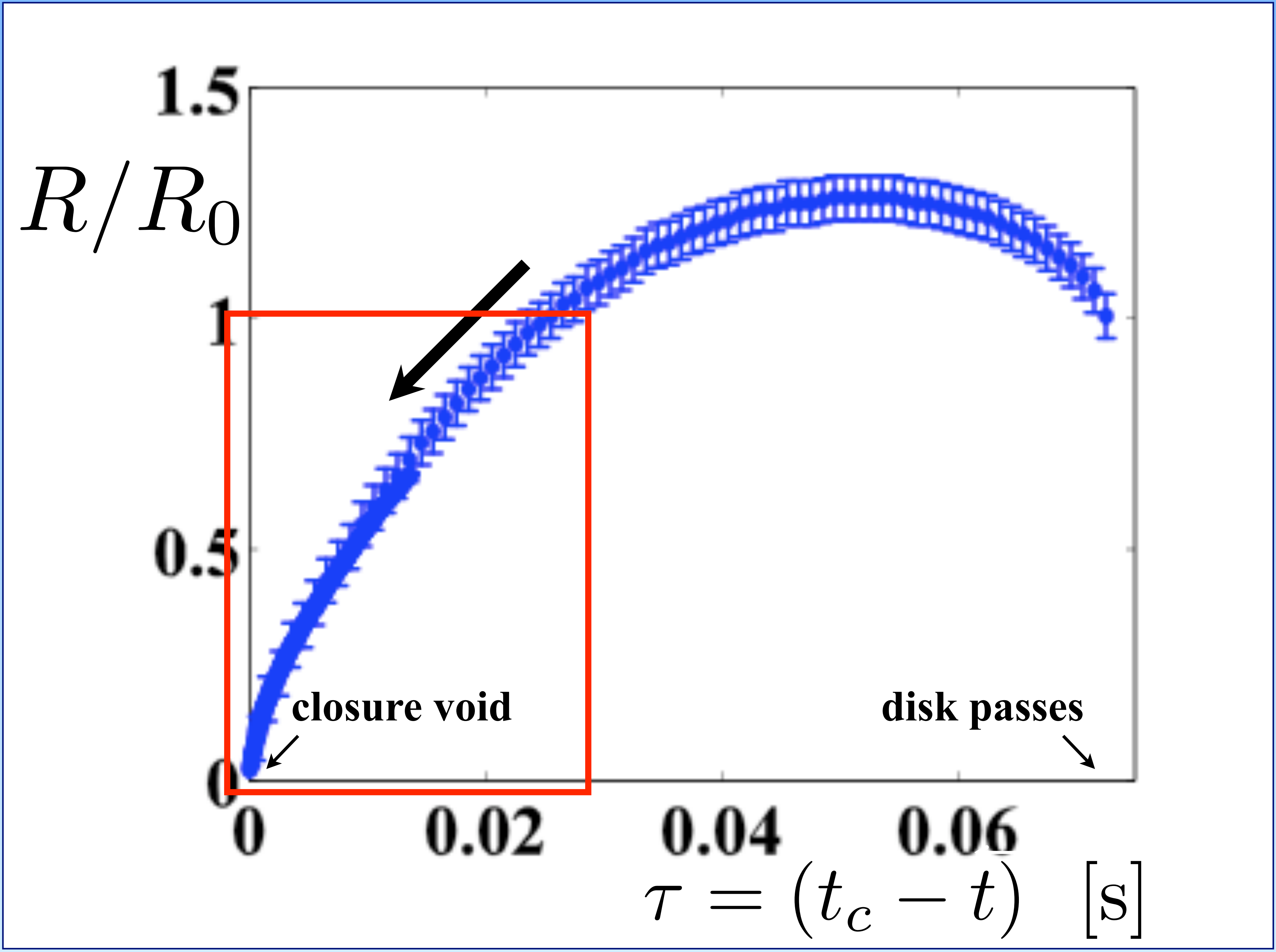
$$\Delta t_{coll}$$

determined by hydrostatic pressure

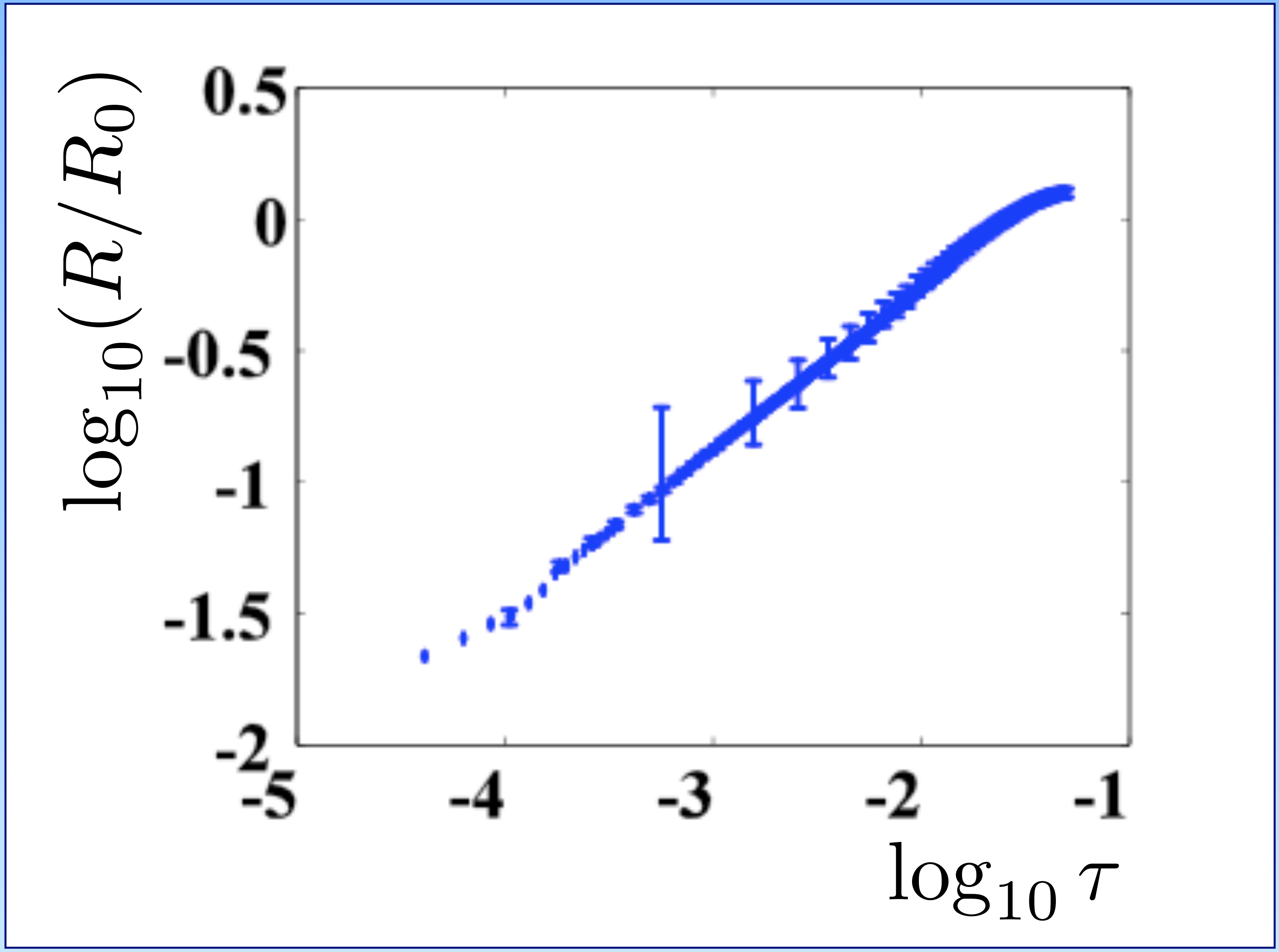
$$\frac{d}{dt}(R\dot{R}) \log \frac{R}{R_\infty} + \frac{1}{2}\dot{R}^2 = gz$$



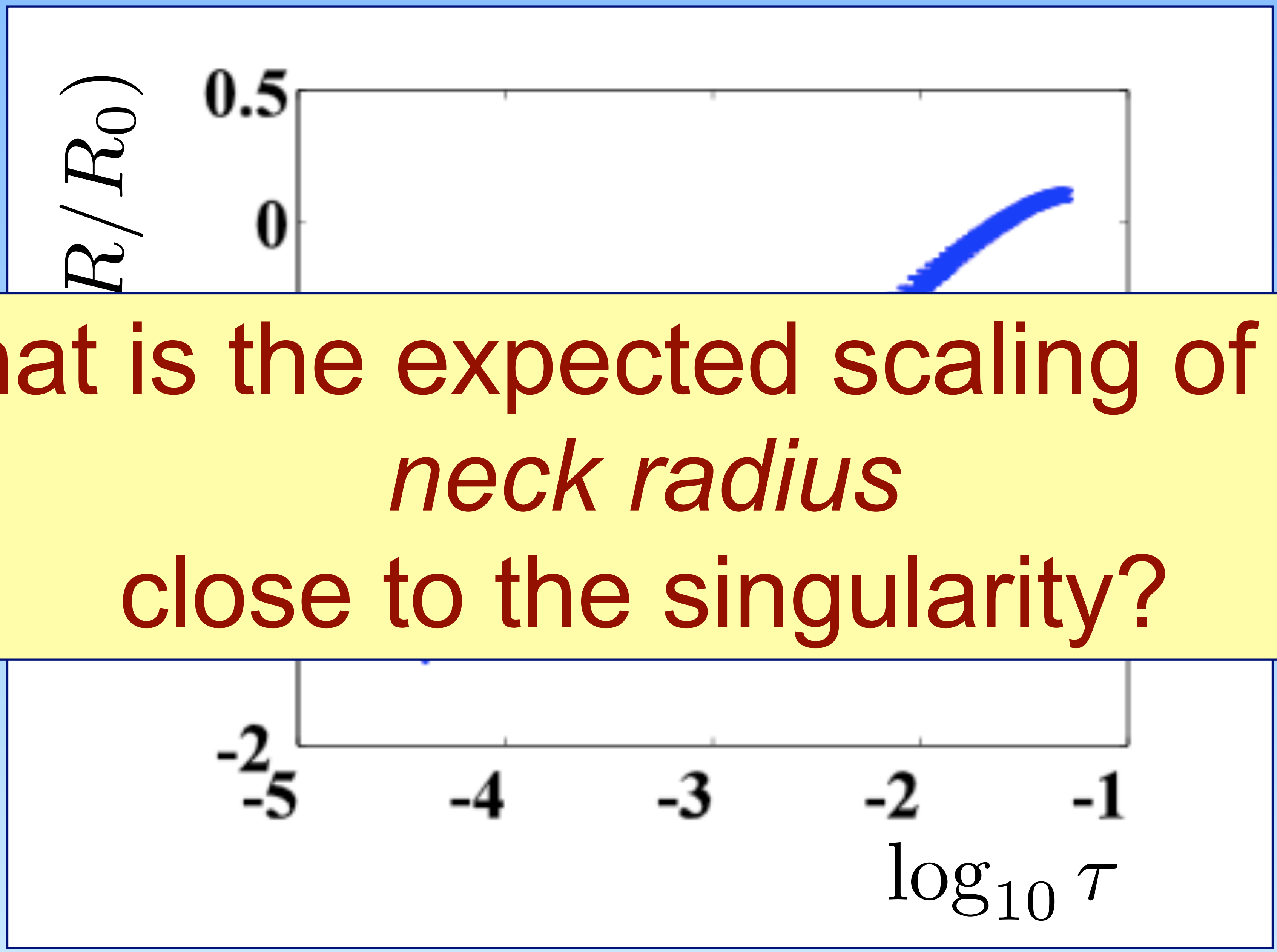
Neck radius $R(\tau)$ [experiment]



Neck radius scaling



Neck radius scaling



What is the expected scaling of the *neck radius* close to the singularity?

Back to 2D Rayleigh

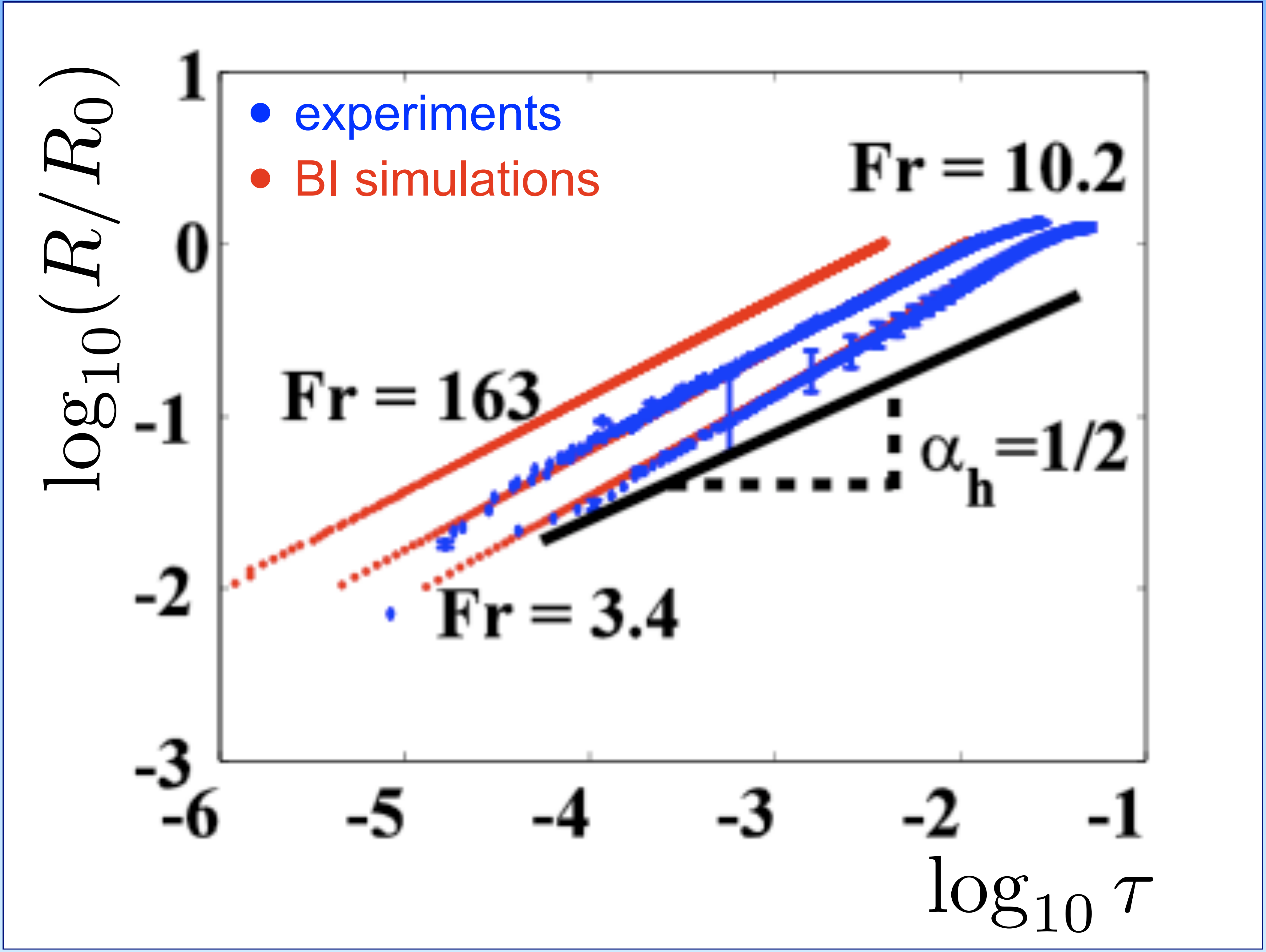
$$\frac{d}{dt}(R\dot{R})\underbrace{\log\frac{R}{R_\infty}}_{\text{large}} + \tfrac{1}{2}\dot{R}^2 = \cancel{gz}$$

gravity negligible

$$\left.\begin{aligned} &\frac{d}{dt}(R\dot{R}) = \frac{d^2}{dt^2}(\tfrac{1}{2}R^2) = 0 \\ &R(t_c) = 0 \qquad \text{(collapse time } t_c\text{)} \end{aligned}\right\} \Rightarrow$$

$$\boxed{R(t) \sim R_0\,(t_c - t)^{1/2}}$$

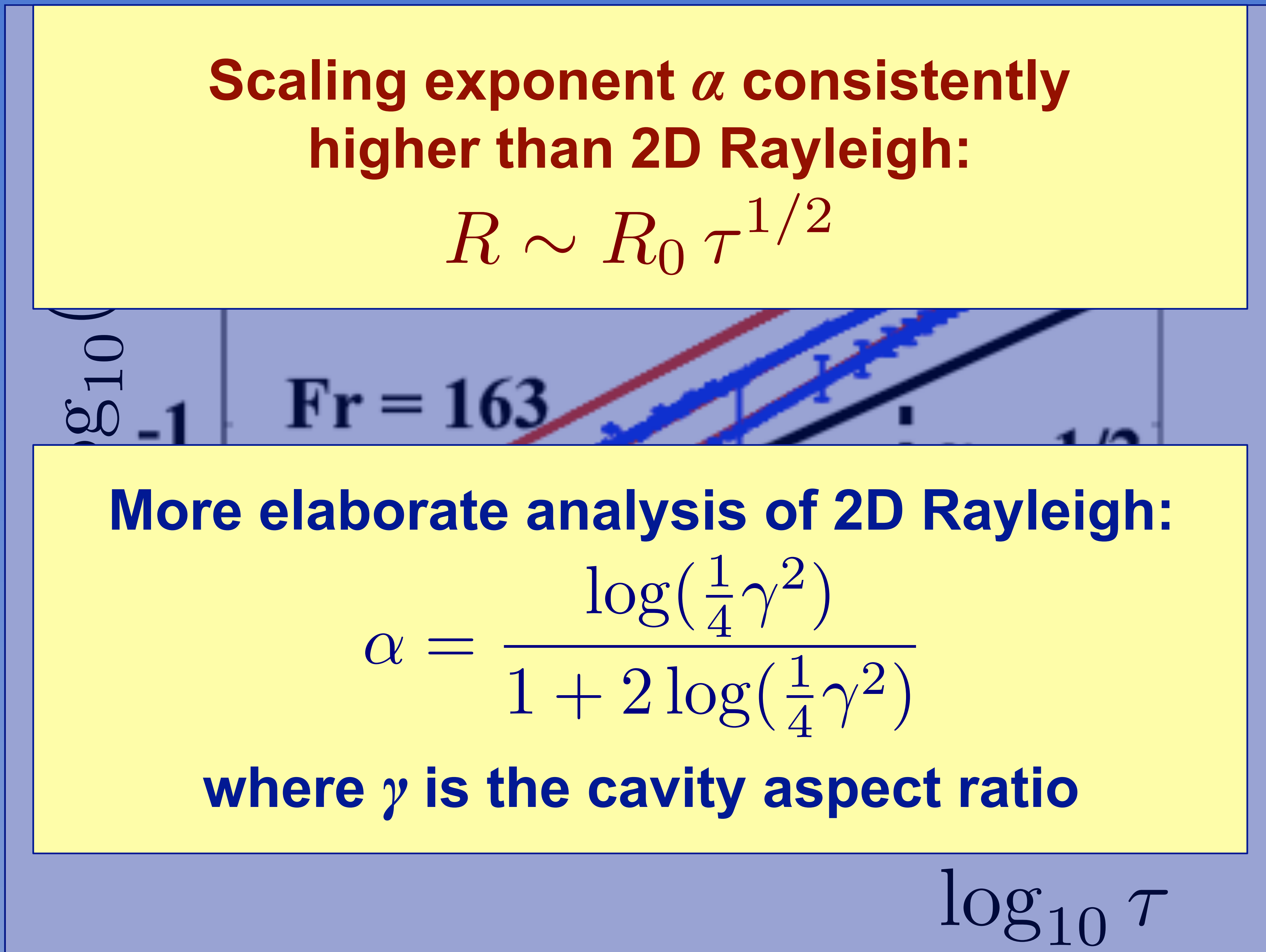
Rayleigh scaling in experiment?



Rayleigh scaling in experiment?

Scaling exponent α consistently higher than 2D Rayleigh:

$$R \sim R_0\, \tau^{1/2}$$



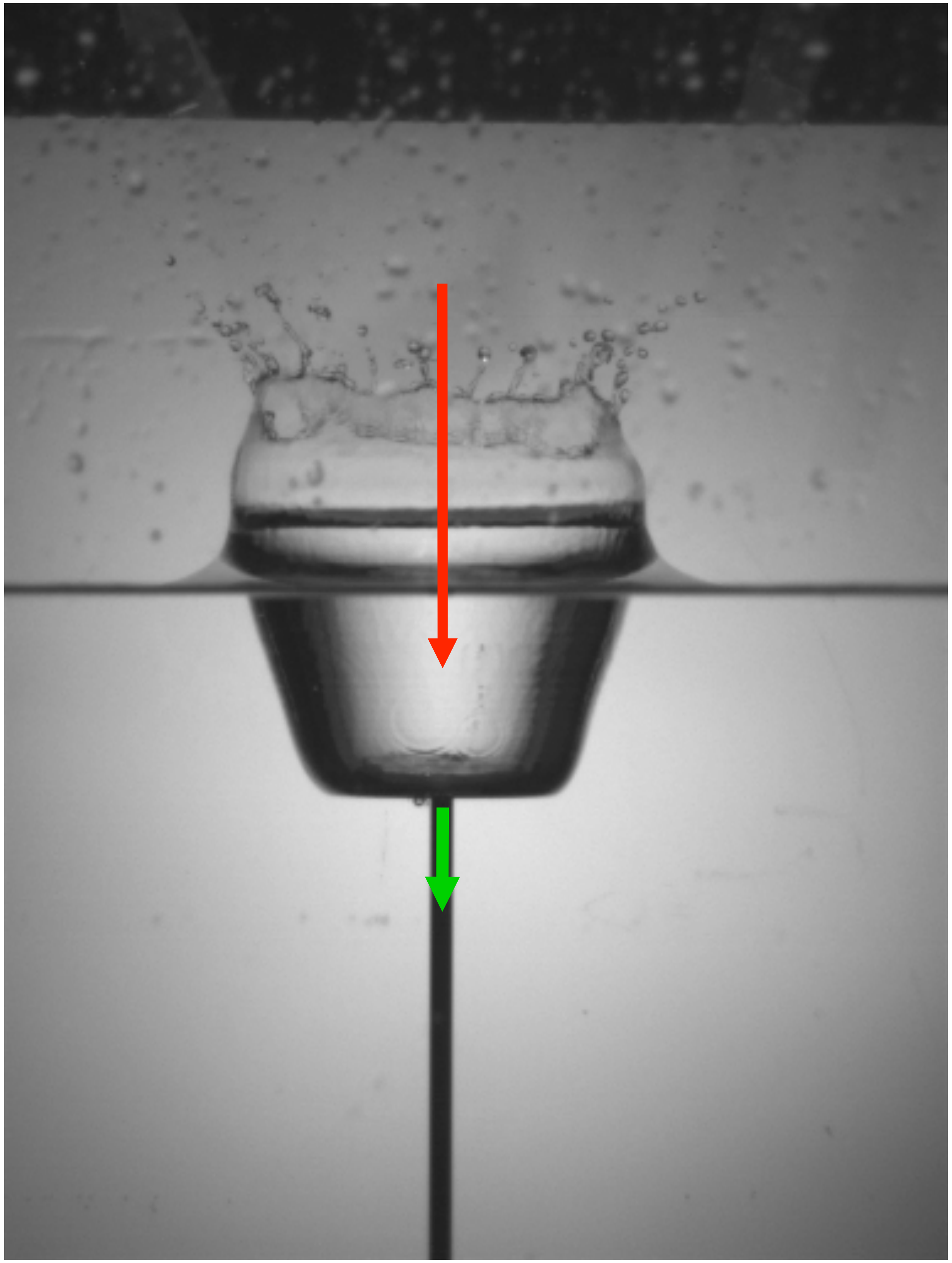
More elaborate analysis of 2D Rayleigh:

$$\alpha = \frac{\log(\frac{1}{4}\gamma^2)}{1 + 2\log(\frac{1}{4}\gamma^2)}$$

where γ is the cavity aspect ratio

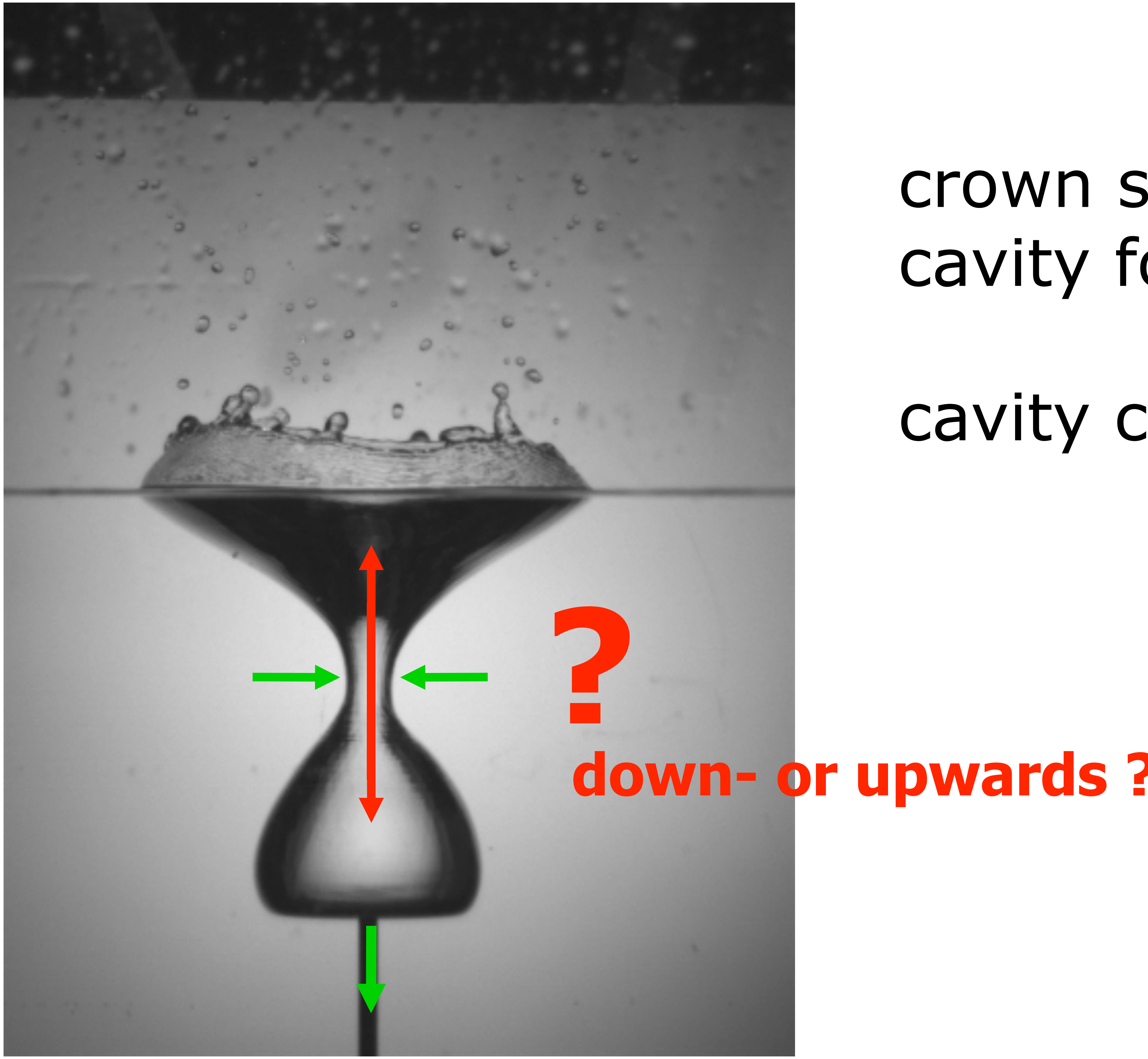
J. M. Gordillo & M. Pérez-Saborid, JFM **562**, 303 (2006); J. Eggers *et al.*, PRL **98**, 094502 (2007); S. Gekle *et al*, PRE **80**, 036305 (2009).

How about the airflow in the cavity ?



crown splash &
cavity formation

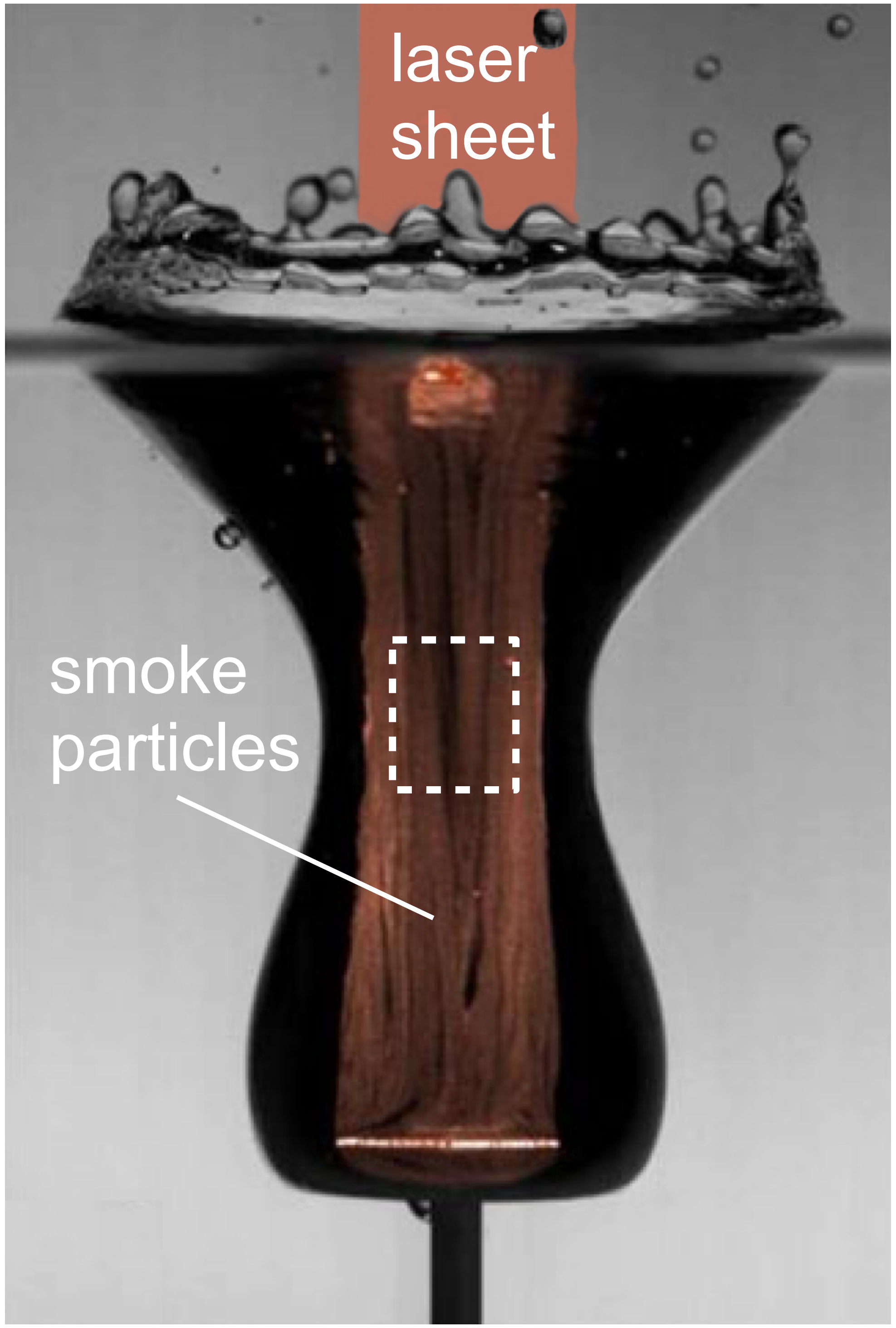
How about the airflow in the cavity ?

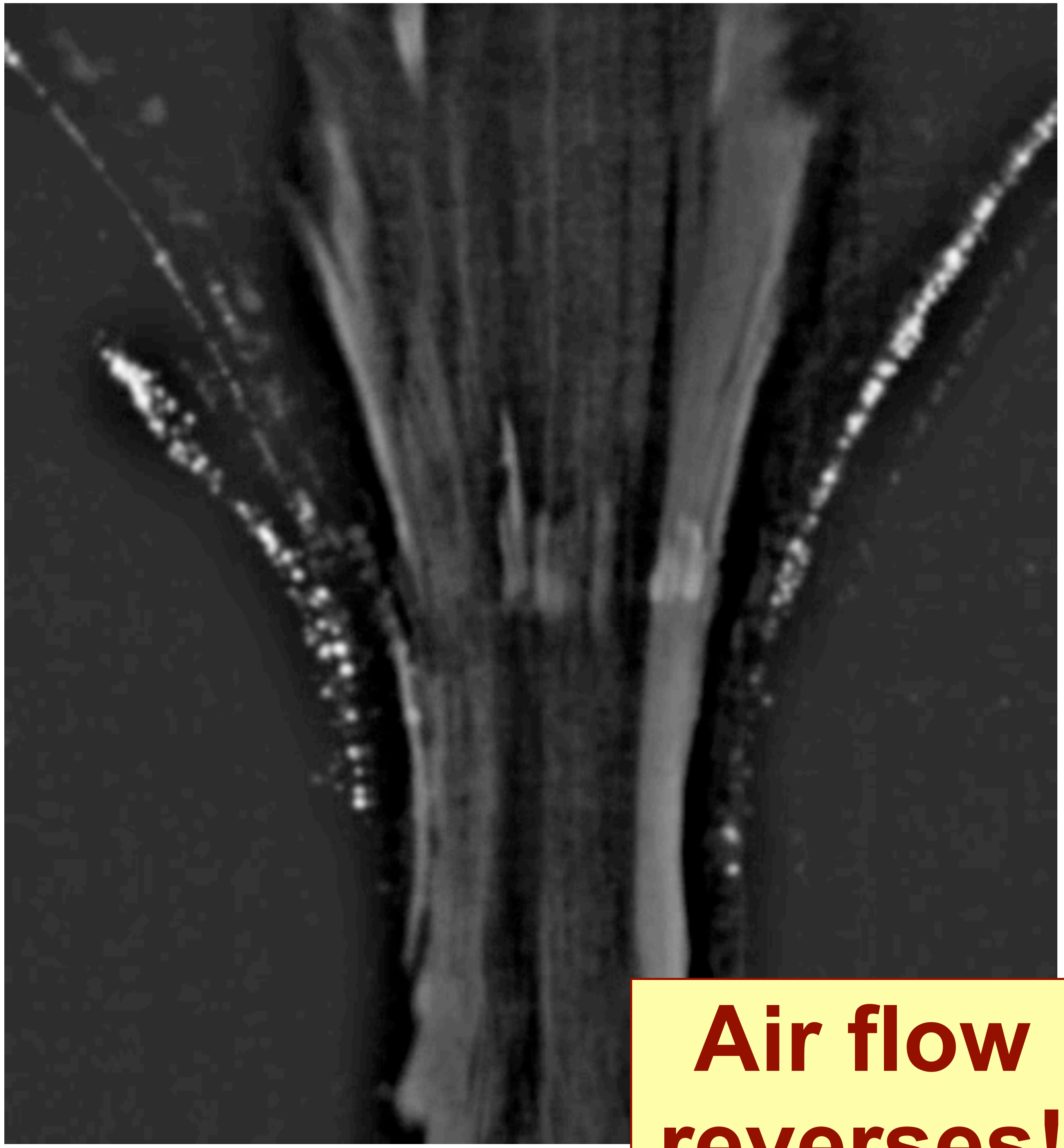


crown splash &
cavity formation

cavity closure

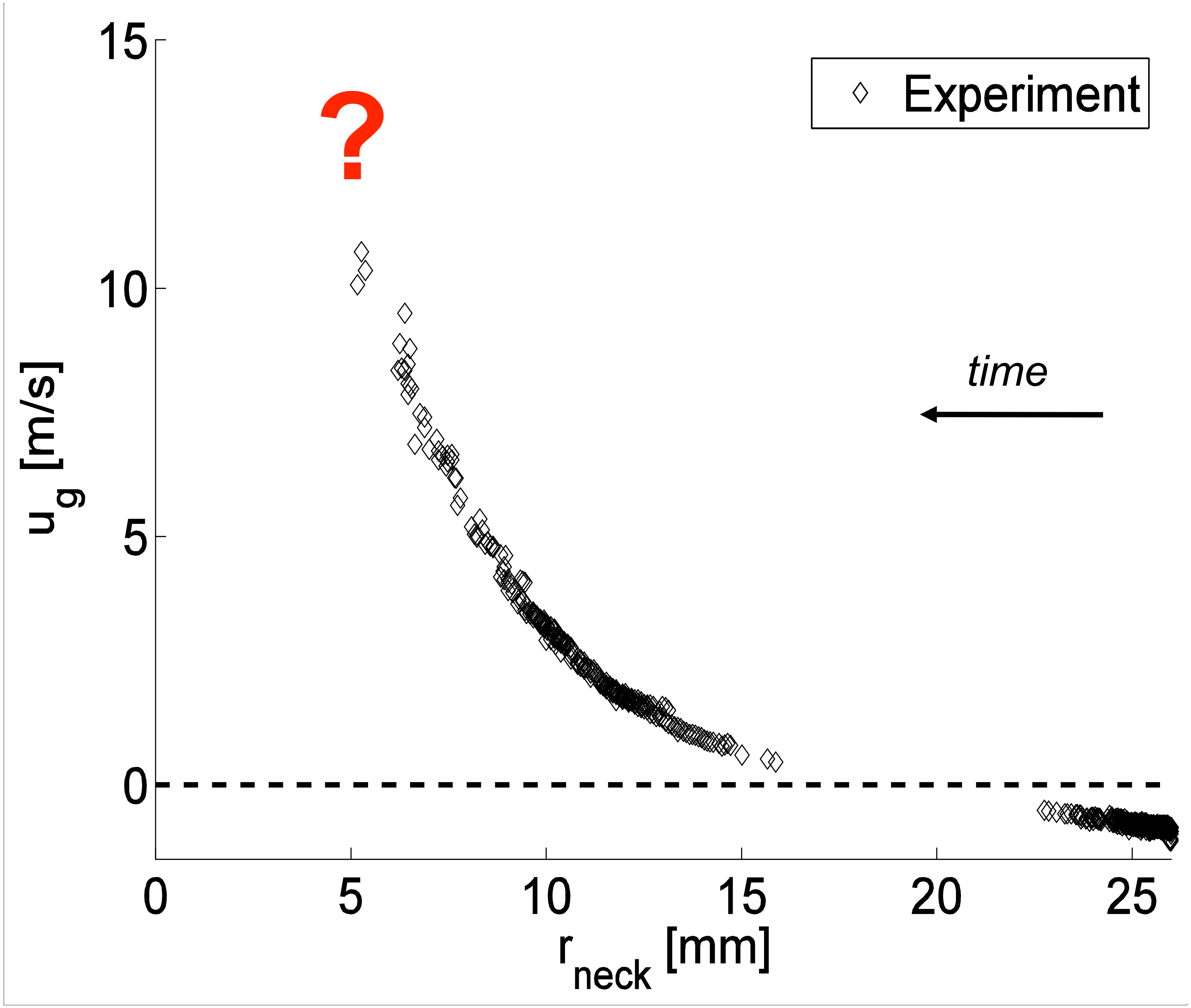
Following smoke particles





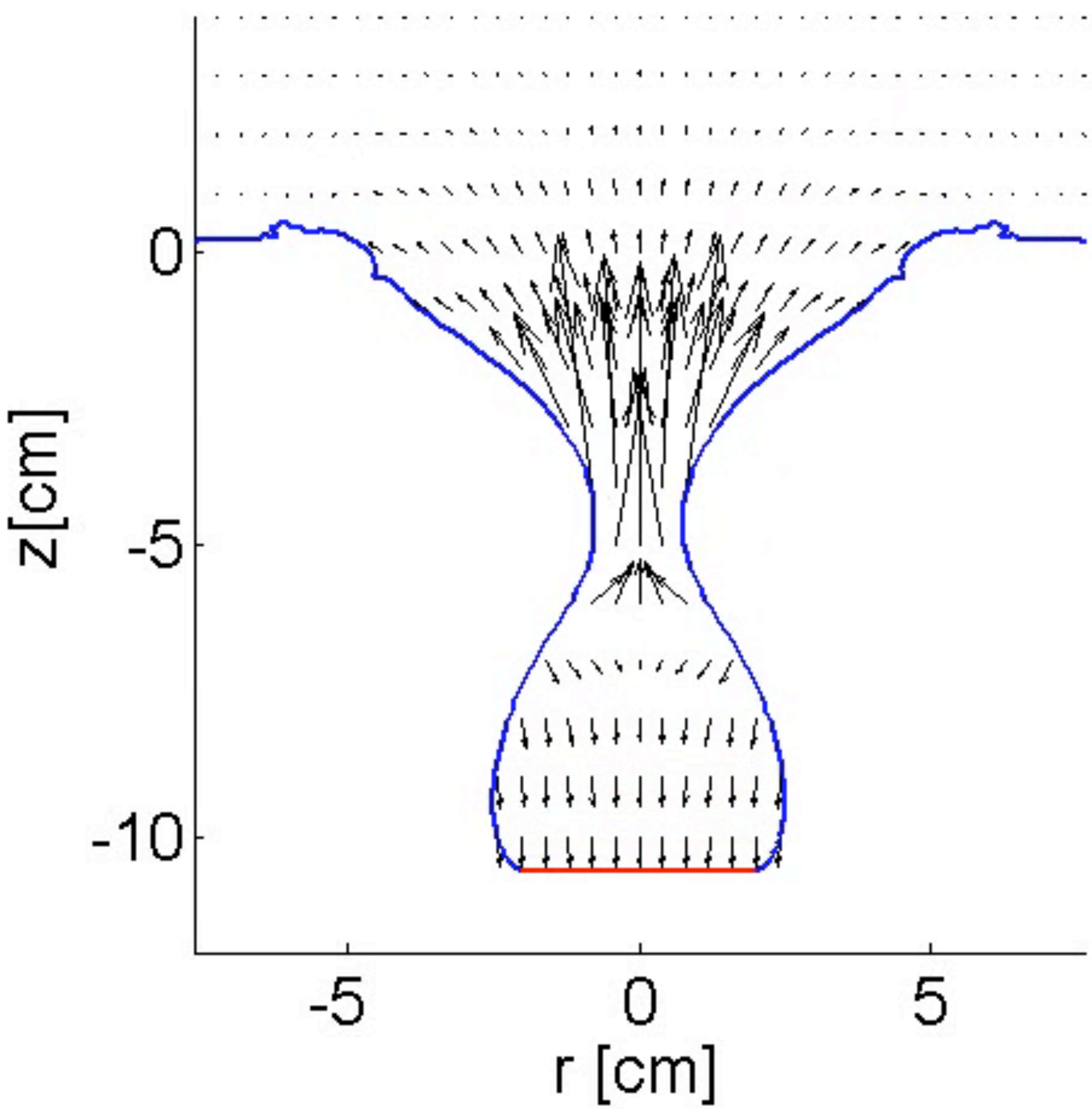
Air flow reverses!

Air speed from smoke measurements



Numerical Modeling 1: Boundary-integral

- Potential flow for both liquid and air:
  irrotational,inviscid, incompressible



Numerical Modeling 1: Boundary-integral

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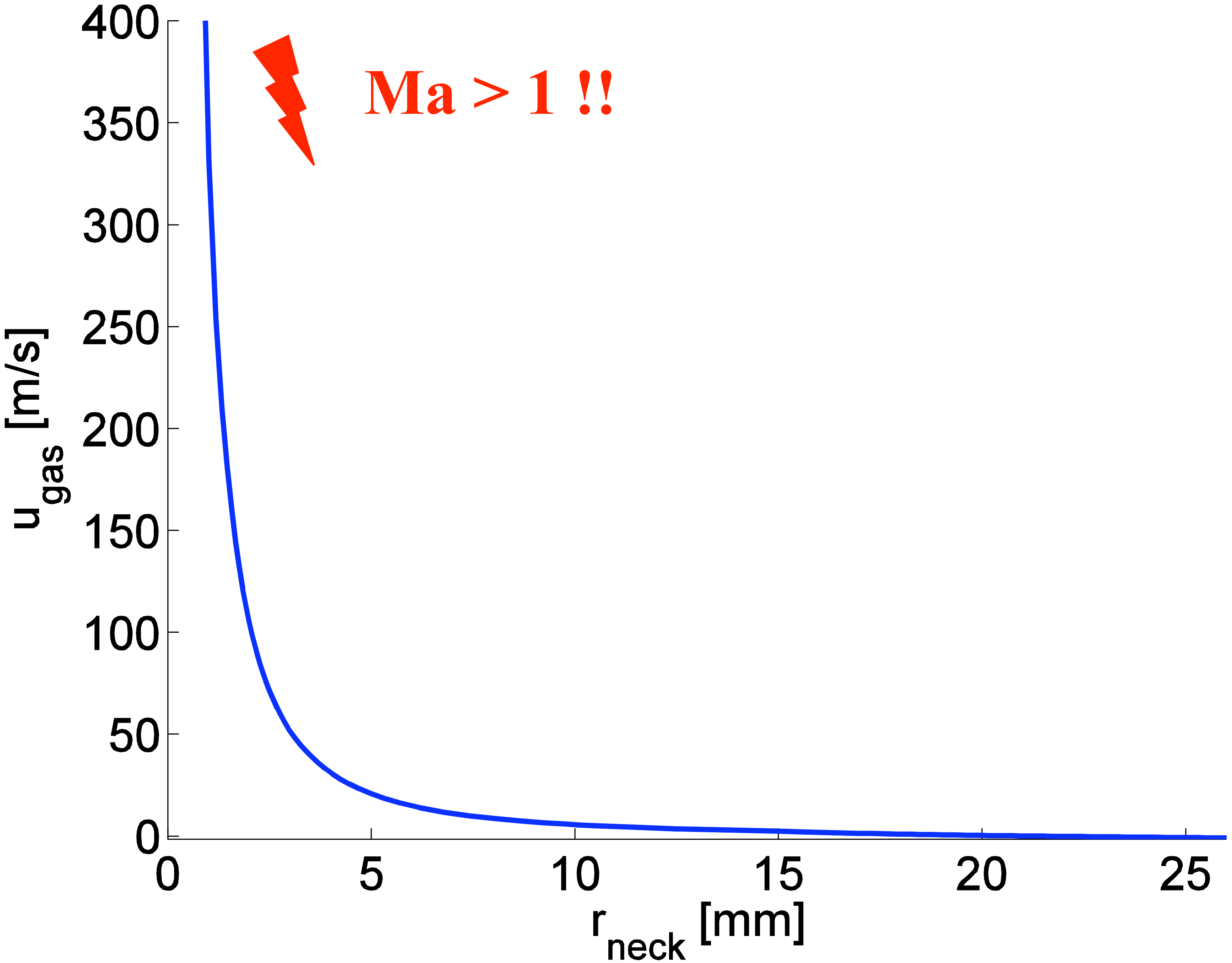
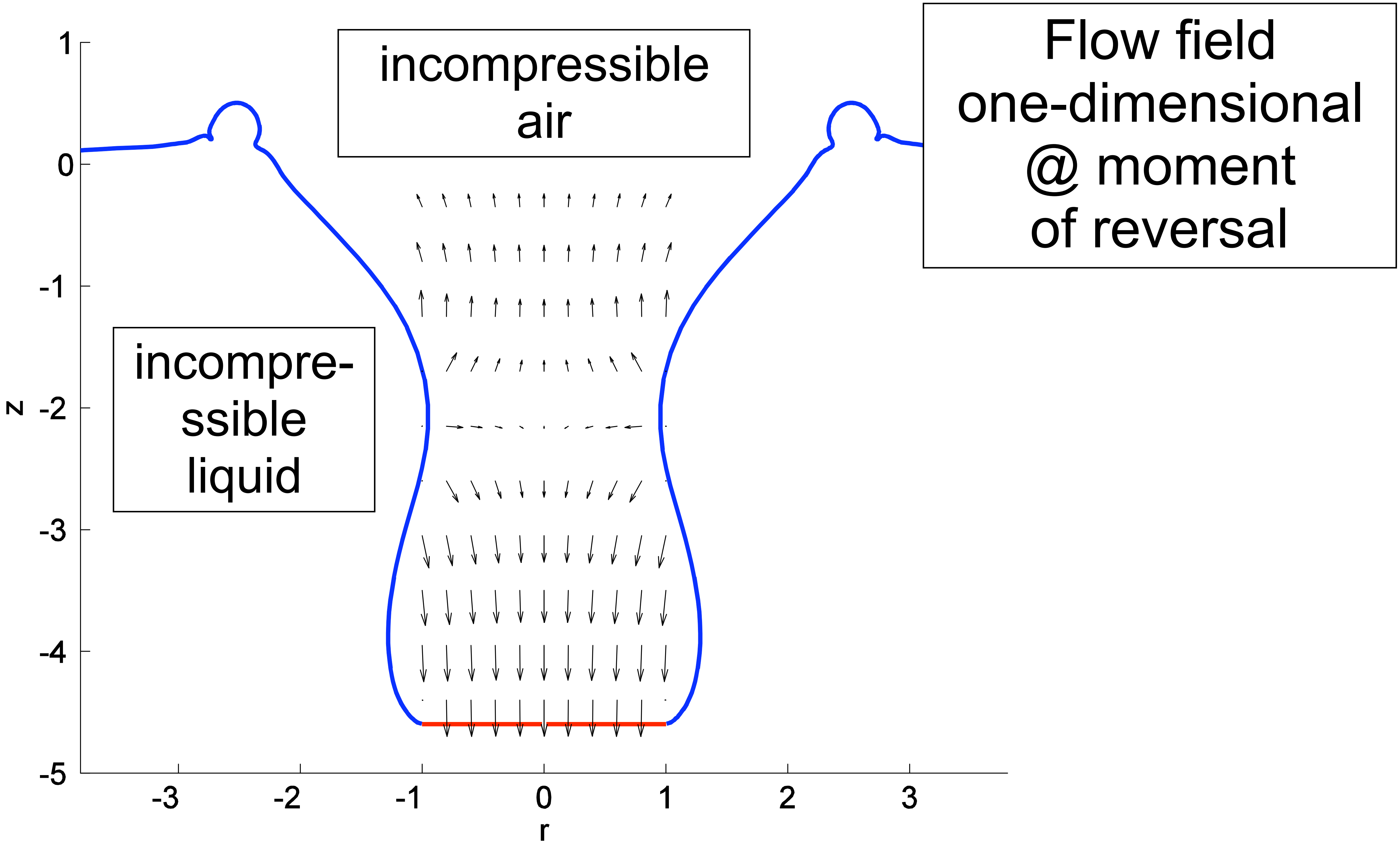
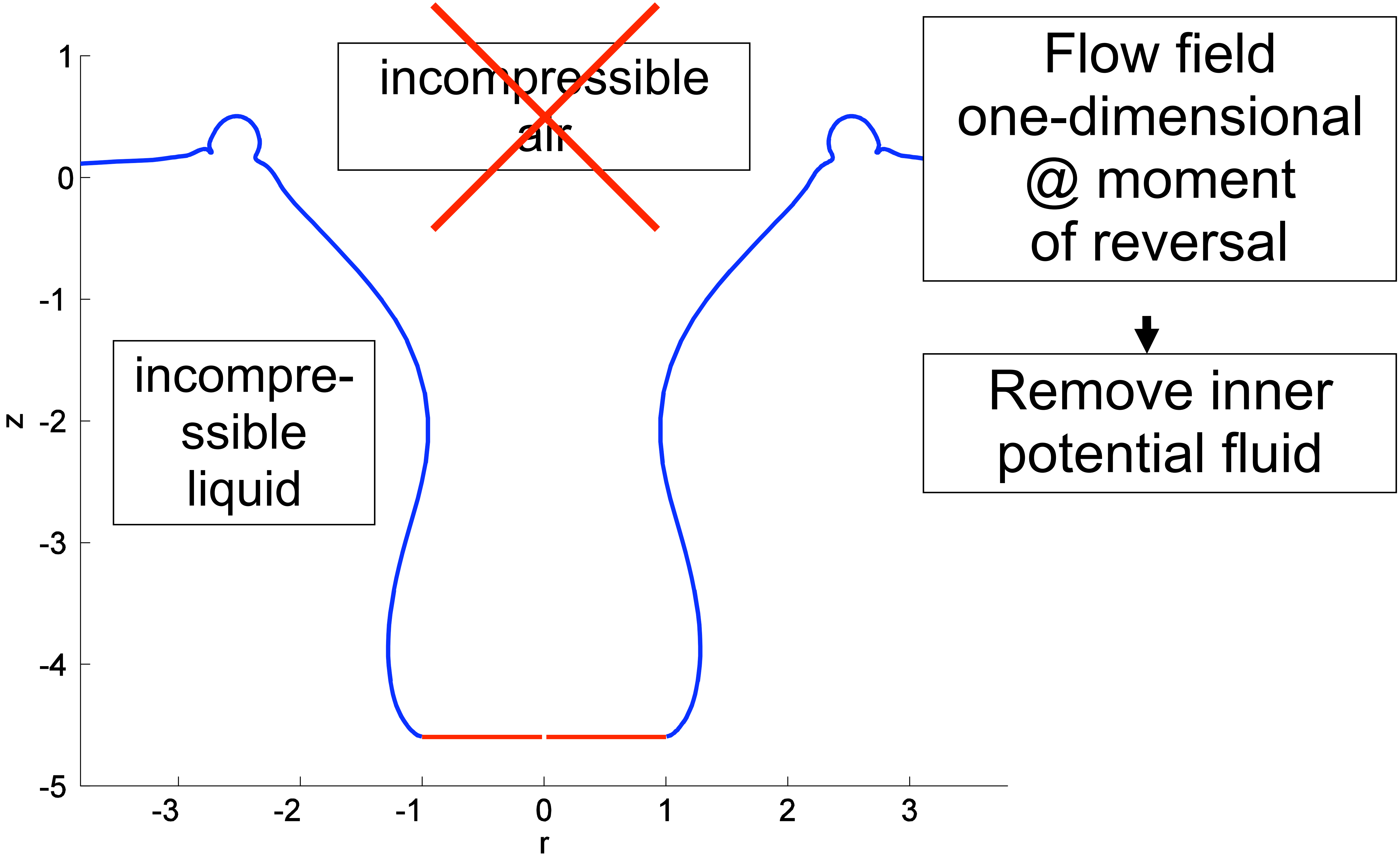


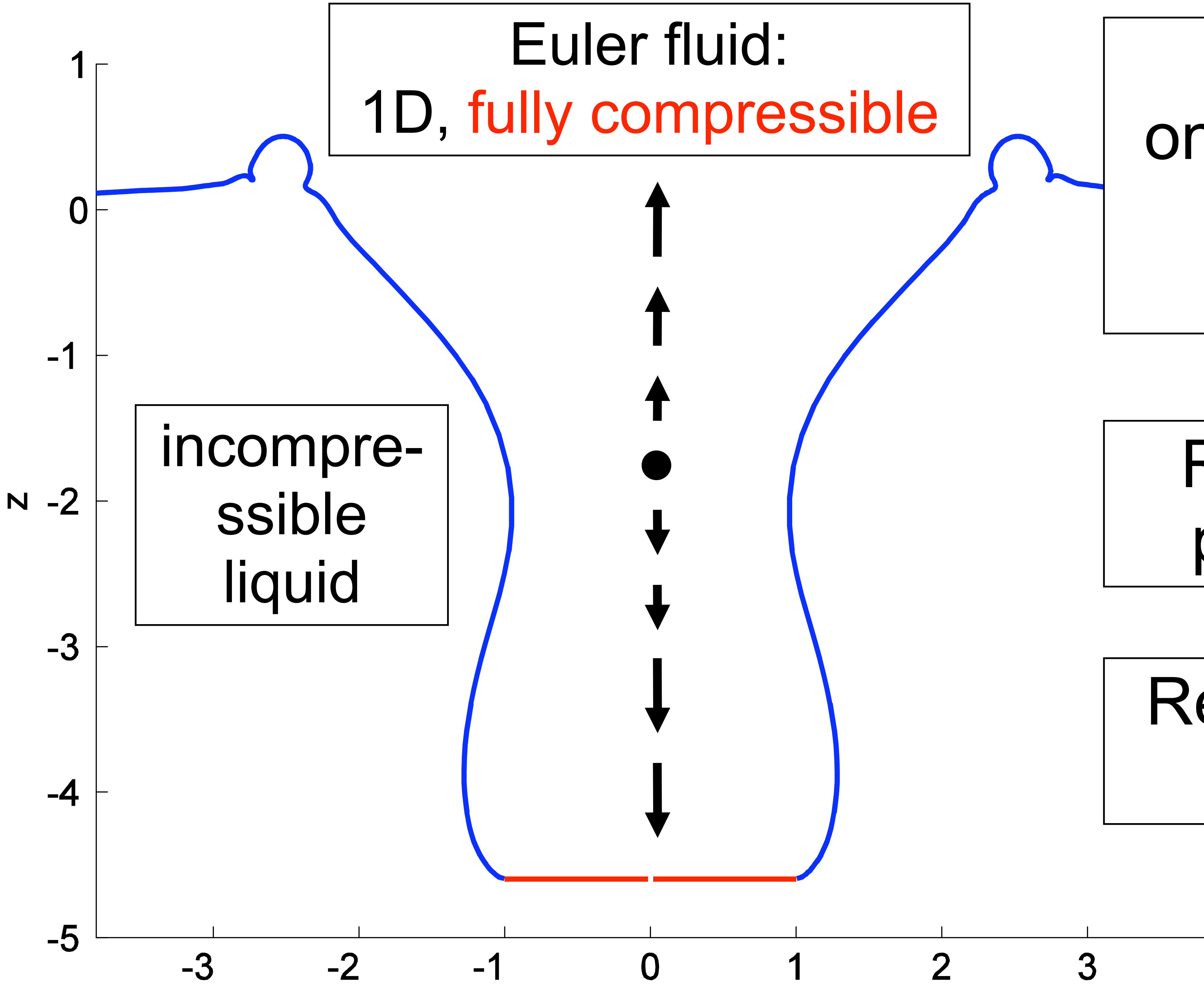
Numerical modeling 2: Multiscale



Numerical modeling 2: Multiscale



Numerical modeling 2: Multiscale



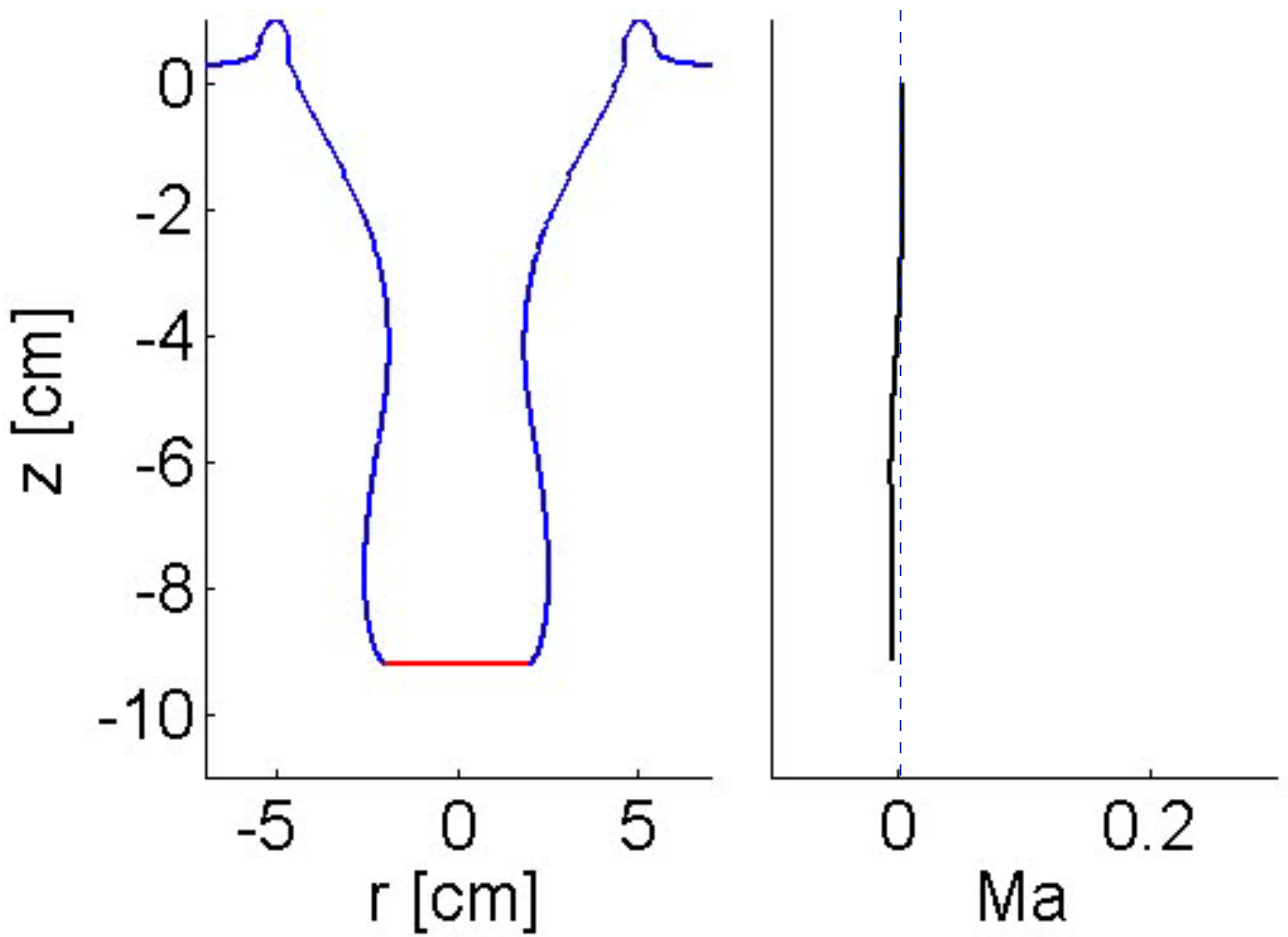
Flow field one-dimensional @ moment of reversal

Remove inner potential fluid

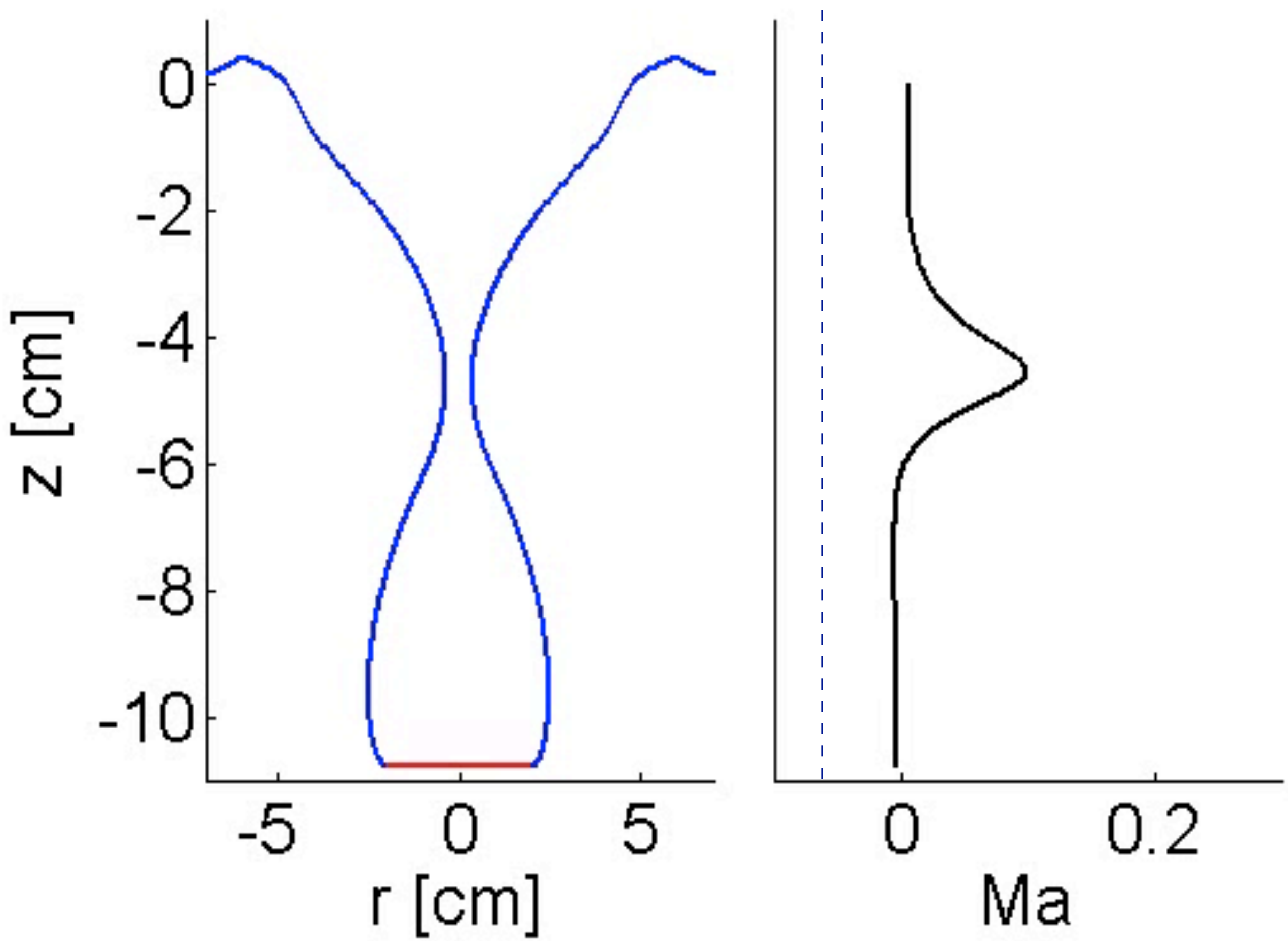
Replace with 1D Euler fluid

Two-way coupling: surface pressure

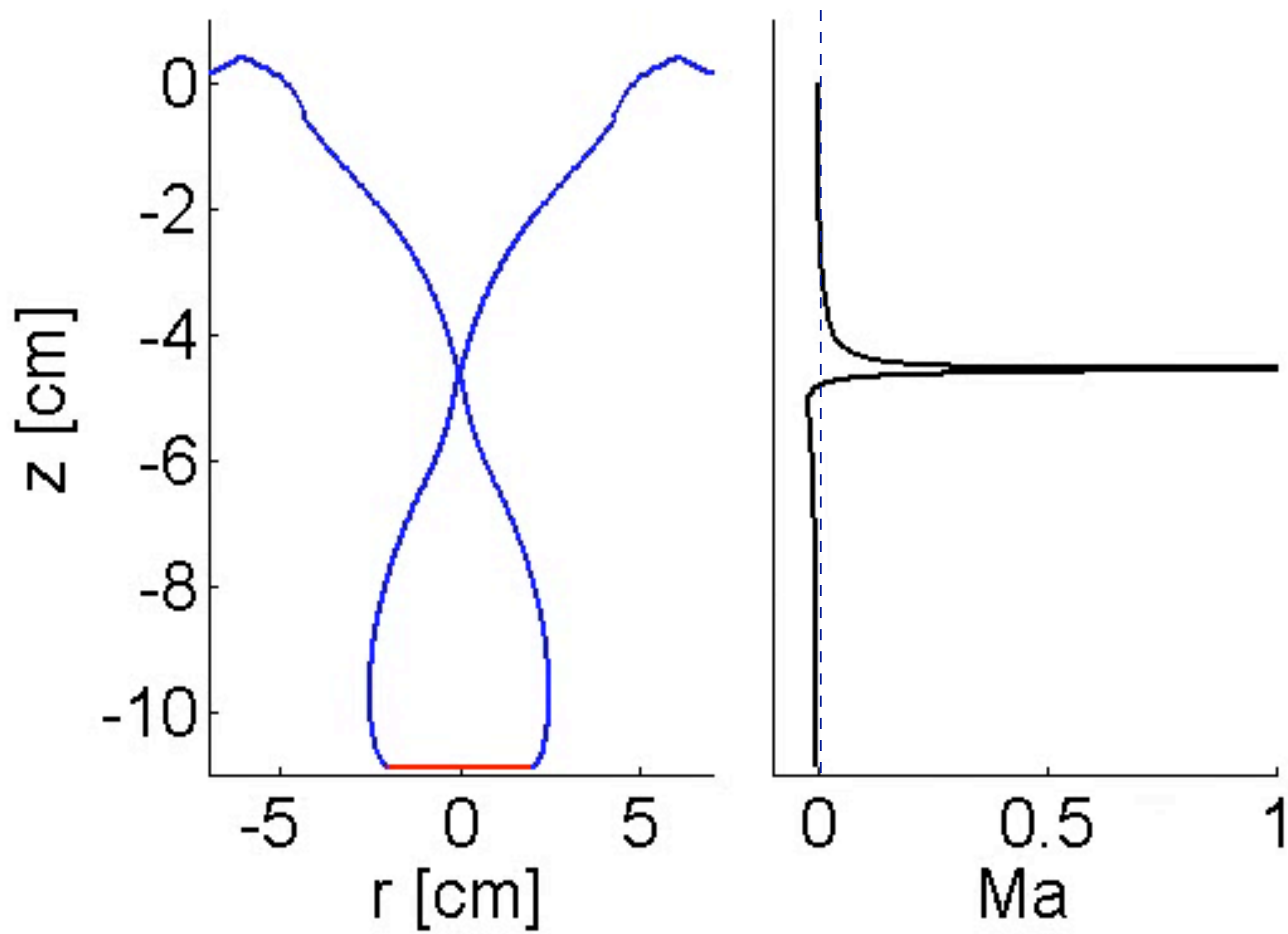
Results: Air velocity profile



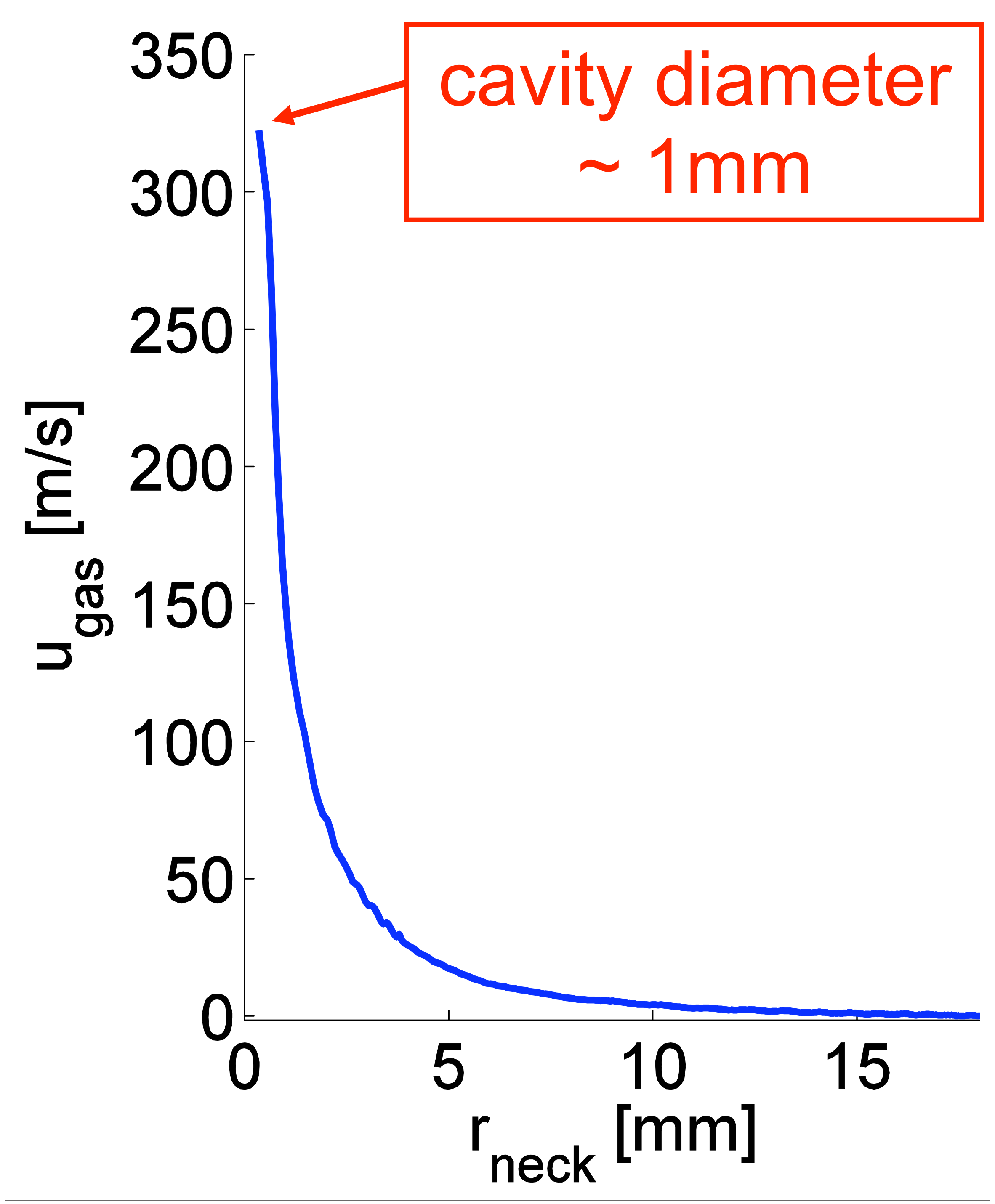
Results: Air velocity profile

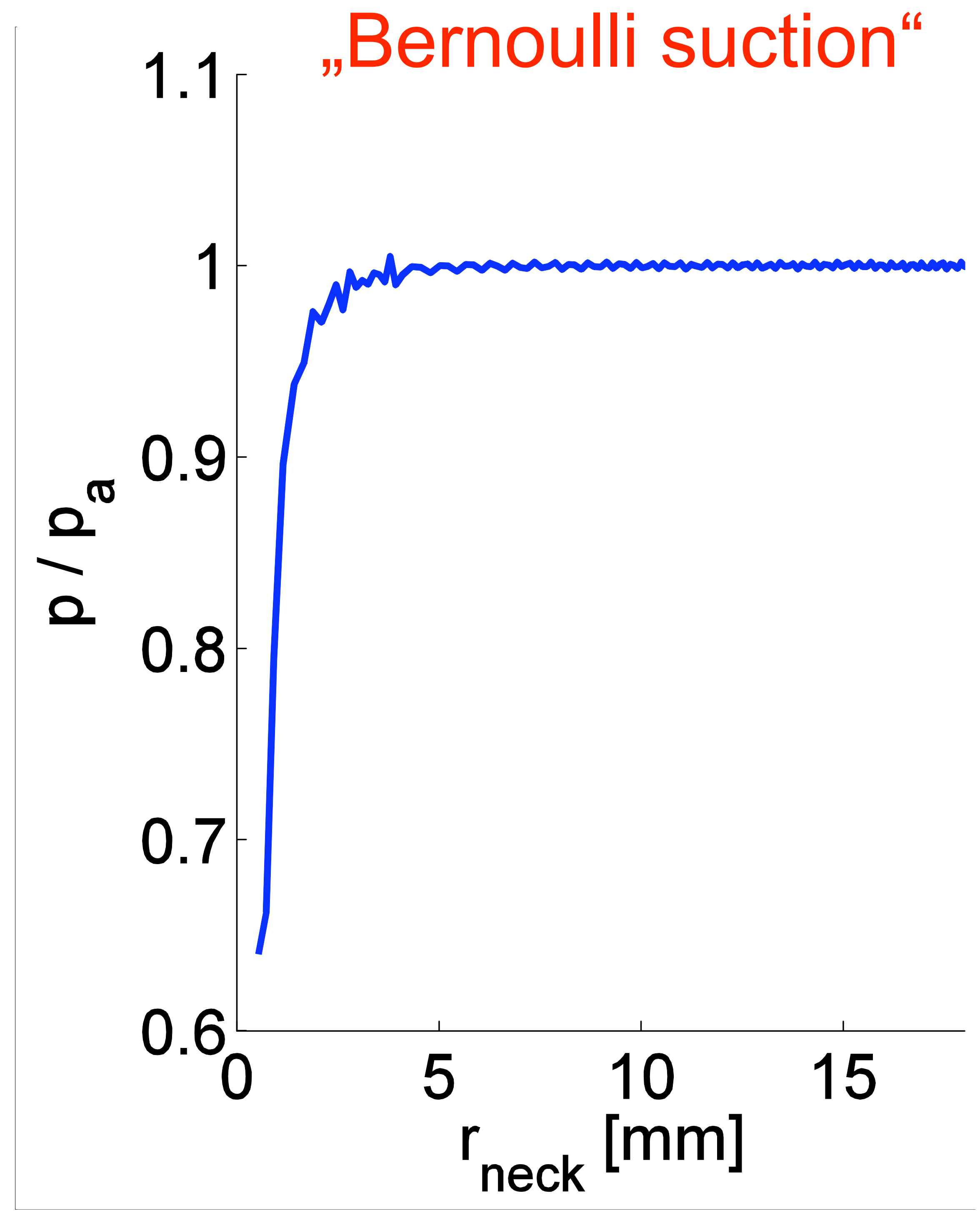


Results: Air velocity profile

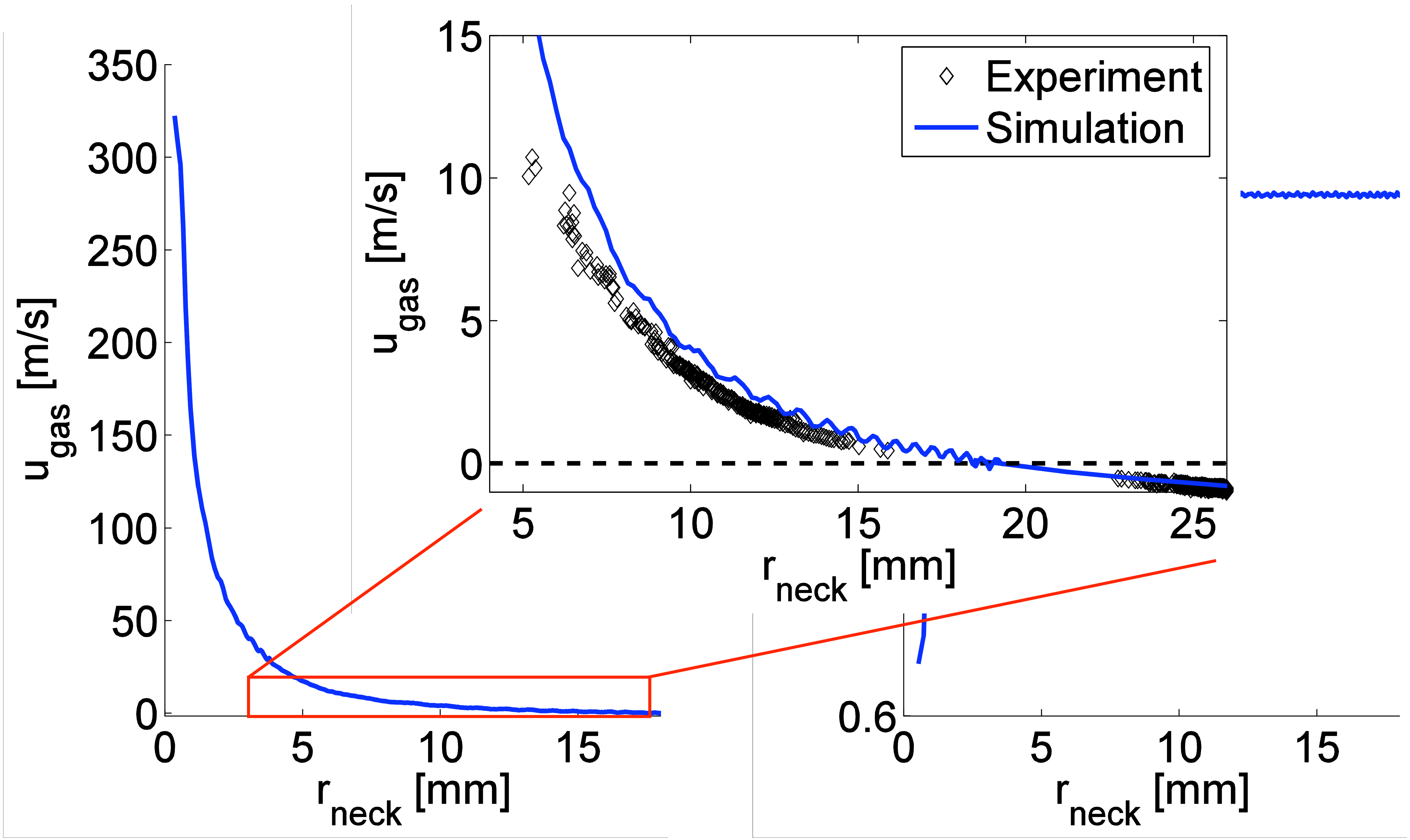


Results: Air flow at the cavity neck





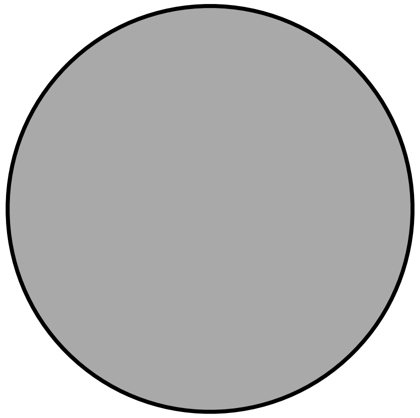
Results: Air flow at the cavity neck



S. Gekle *et al.*, PRL **104**, 024501 (2010).

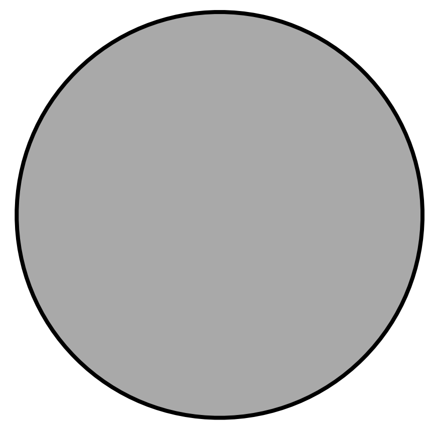
Breaking axial symmetry

Instead of round disc:

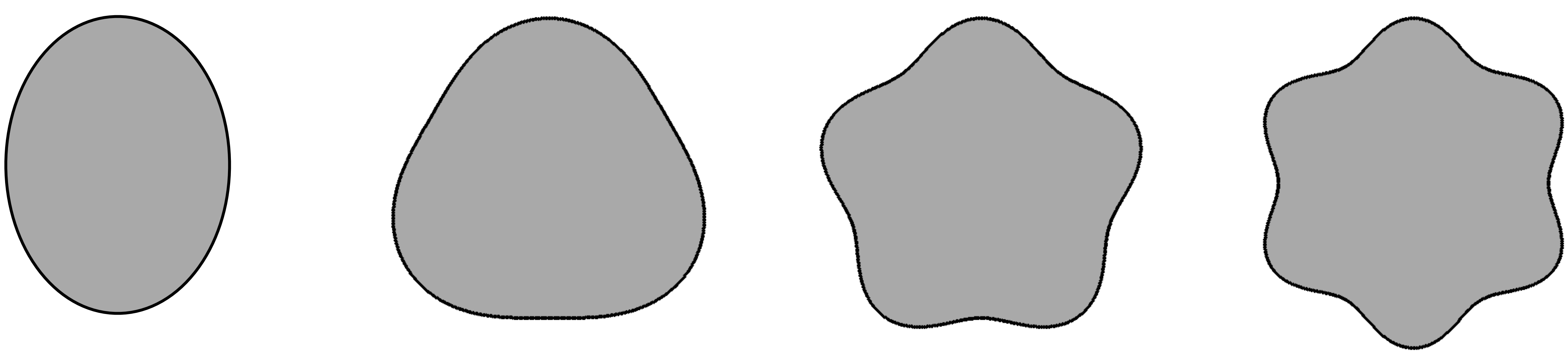


Breaking axial symmetry

Instead of round disc:



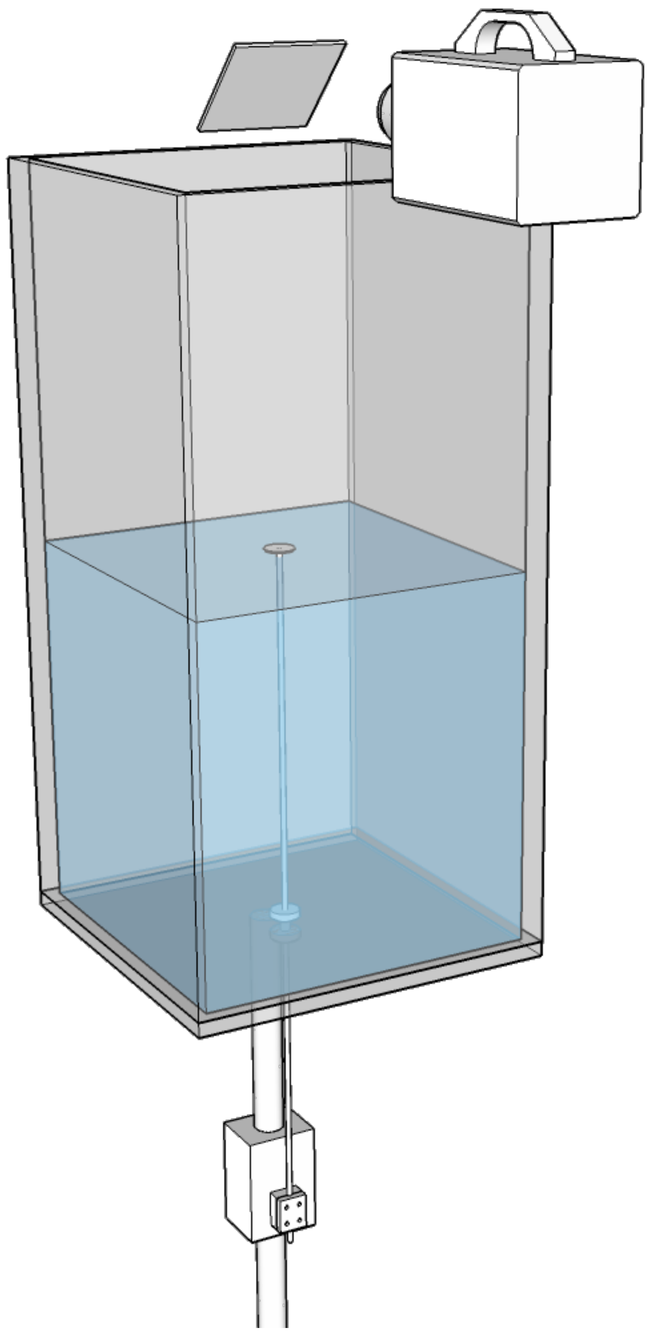
use flowershaped discs with perturbation:



***m* = 2** **_m_ = 3** **_m_ = 5** **_m_ = 6**

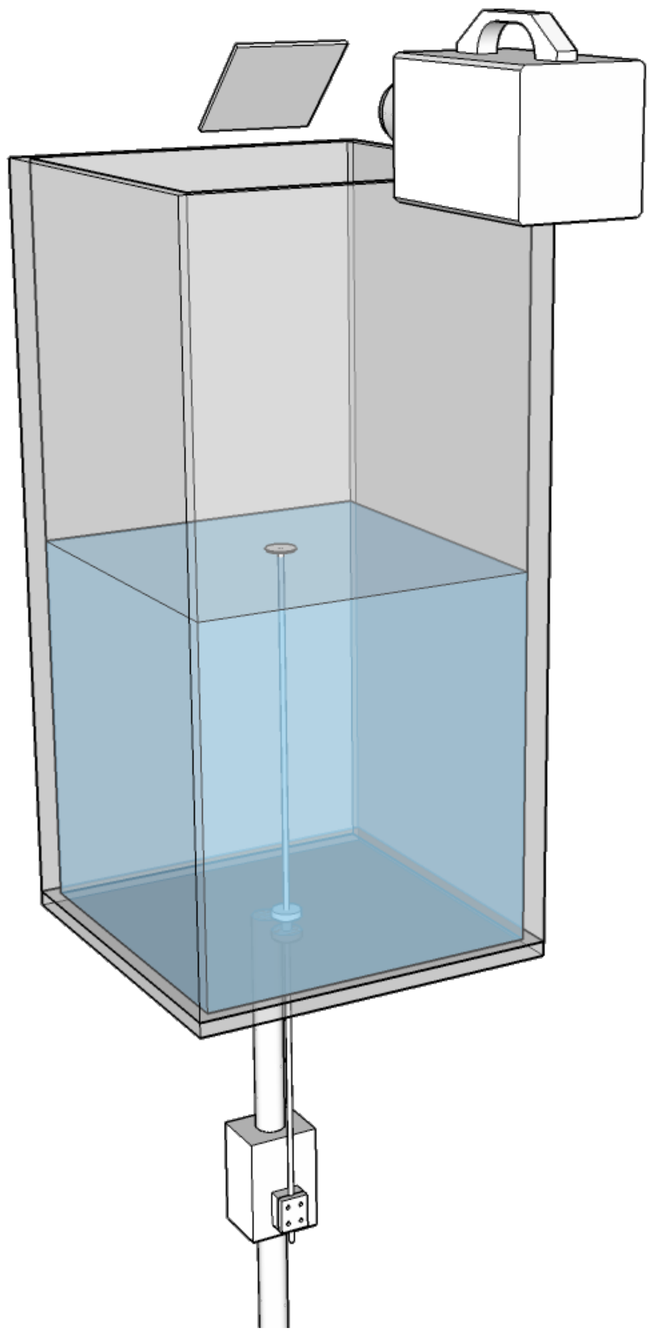
Experiment for m = 2

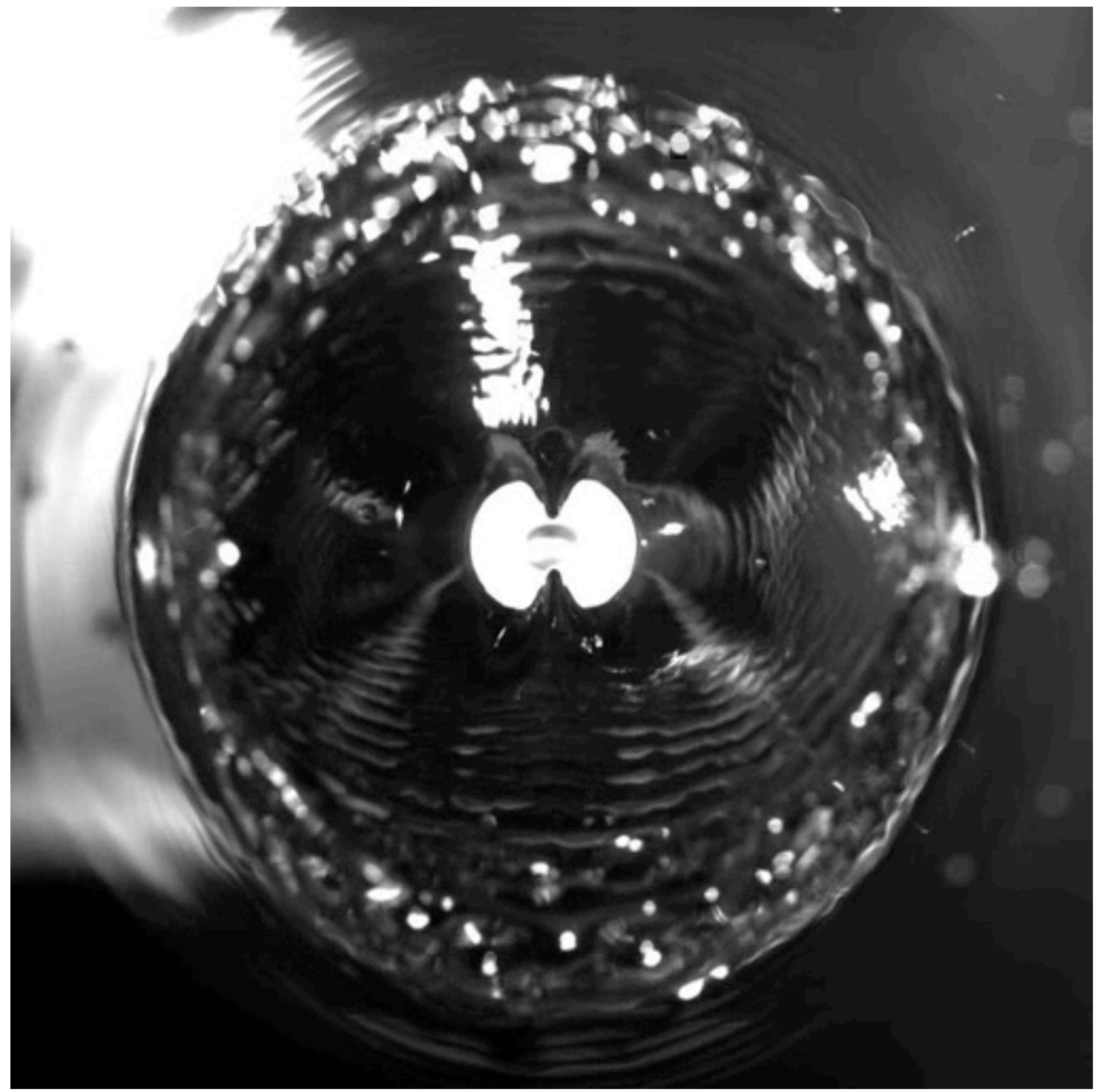
Top view



Experiment for m = 2

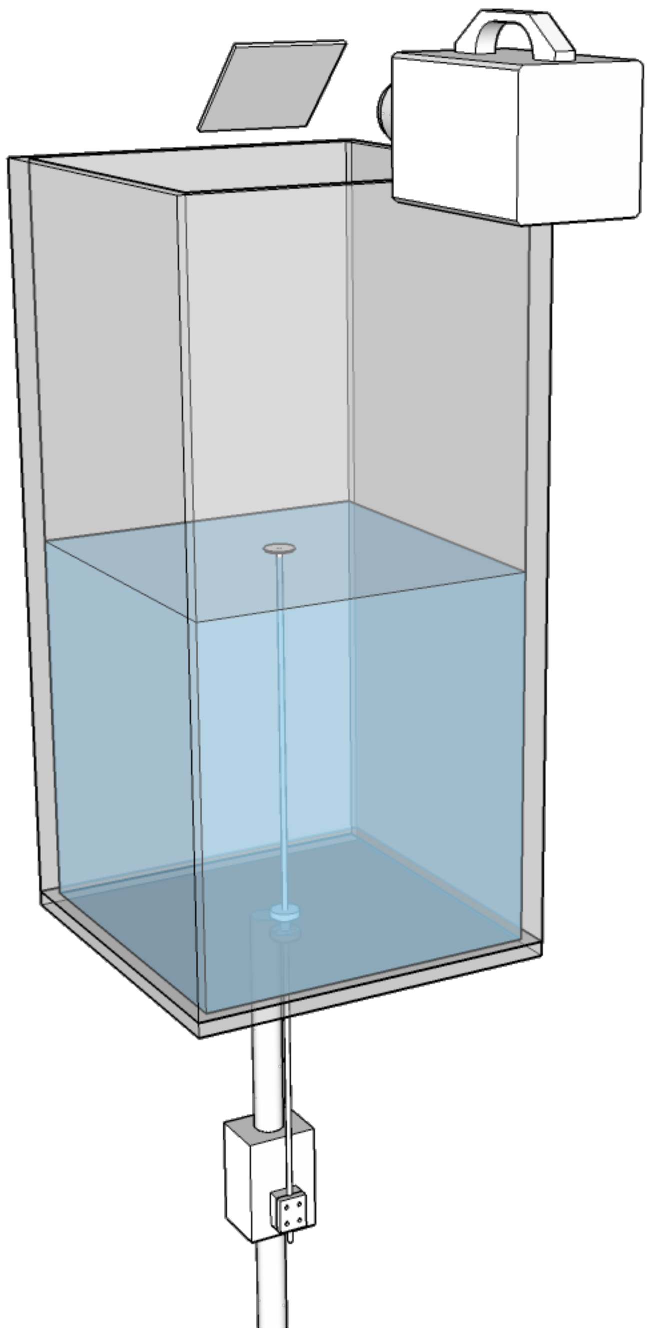
Top view

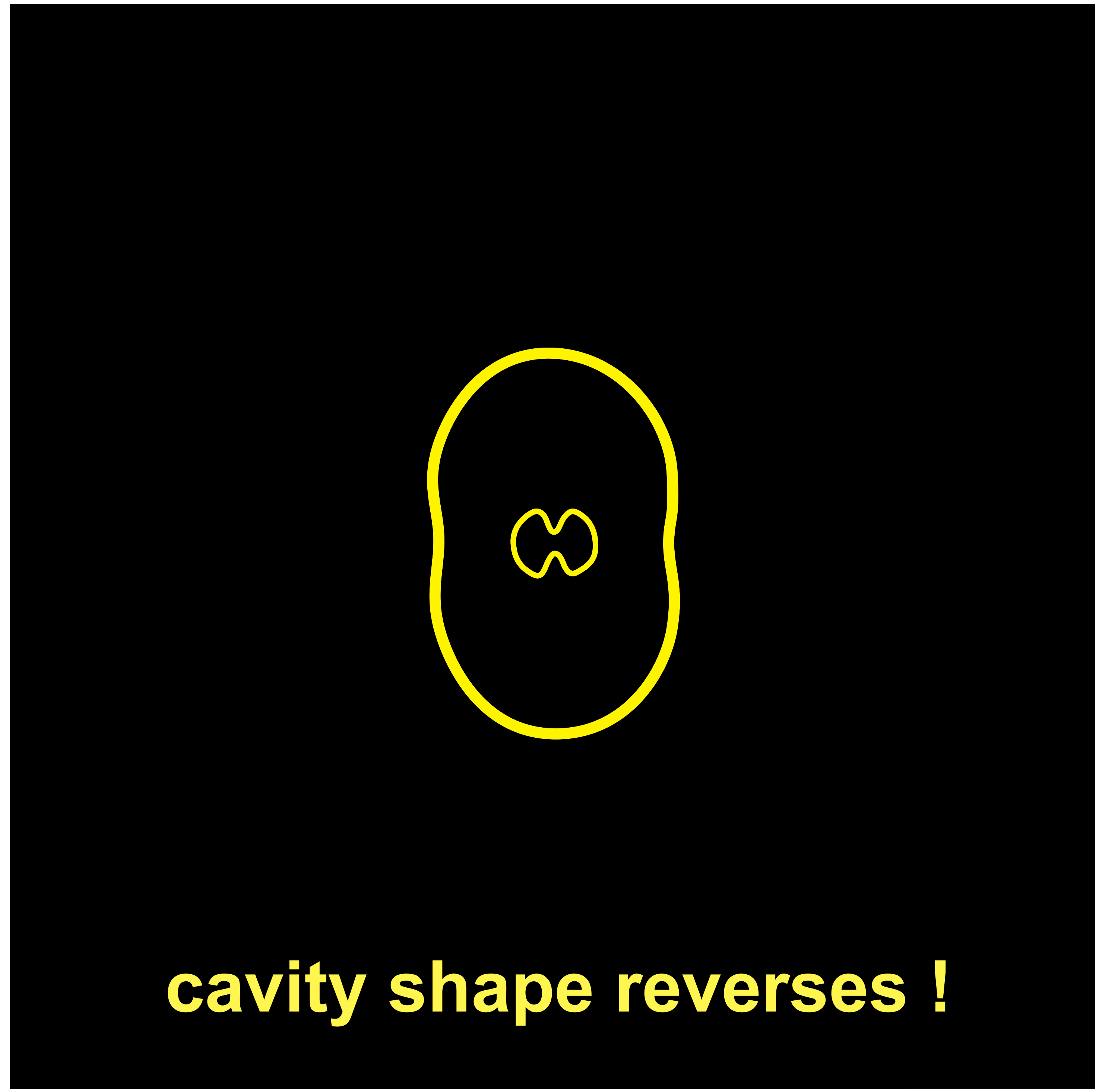




Experiment for $m = 2$

Top view





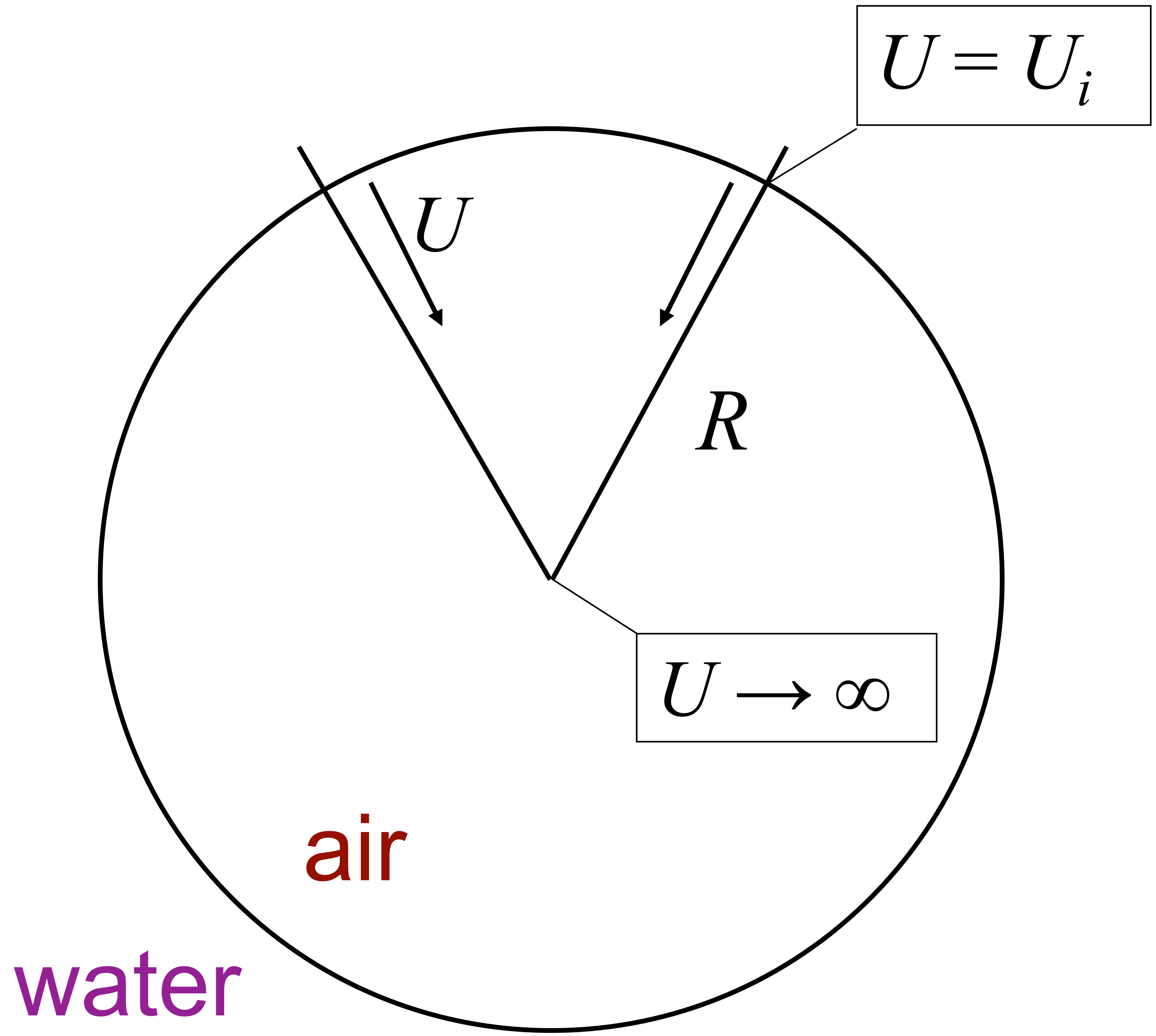
Basic mechanism: circular cavity

Continuity argument:
(cylindrical cavity)

$$U R = C \quad \Rightarrow$$

$$\frac{dU}{dt} = \frac{U^2}{R}$$

$$\left(\dot{U} = -\frac{U\dot{R}}{R} \underset{\substack{\uparrow \\ \dot{R} = -U}}{=} \frac{U^2}{R} \right)$$



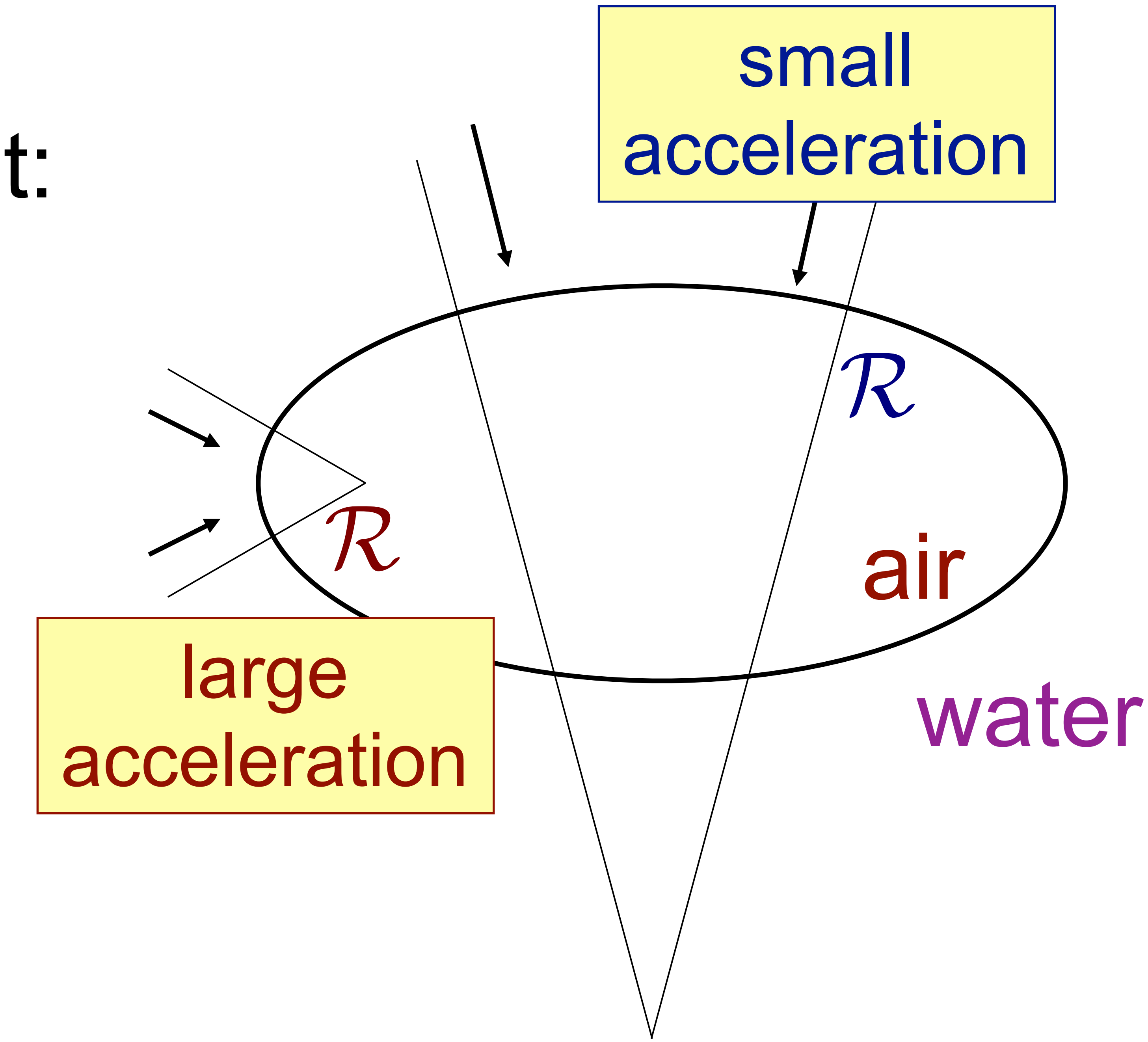
Basic mechanism: elliptical cavity

Continuity argument:
(elliptical cavity)

$$U\mathcal{R} = C$$

$\mathcal{R} =$ local radius of curvature

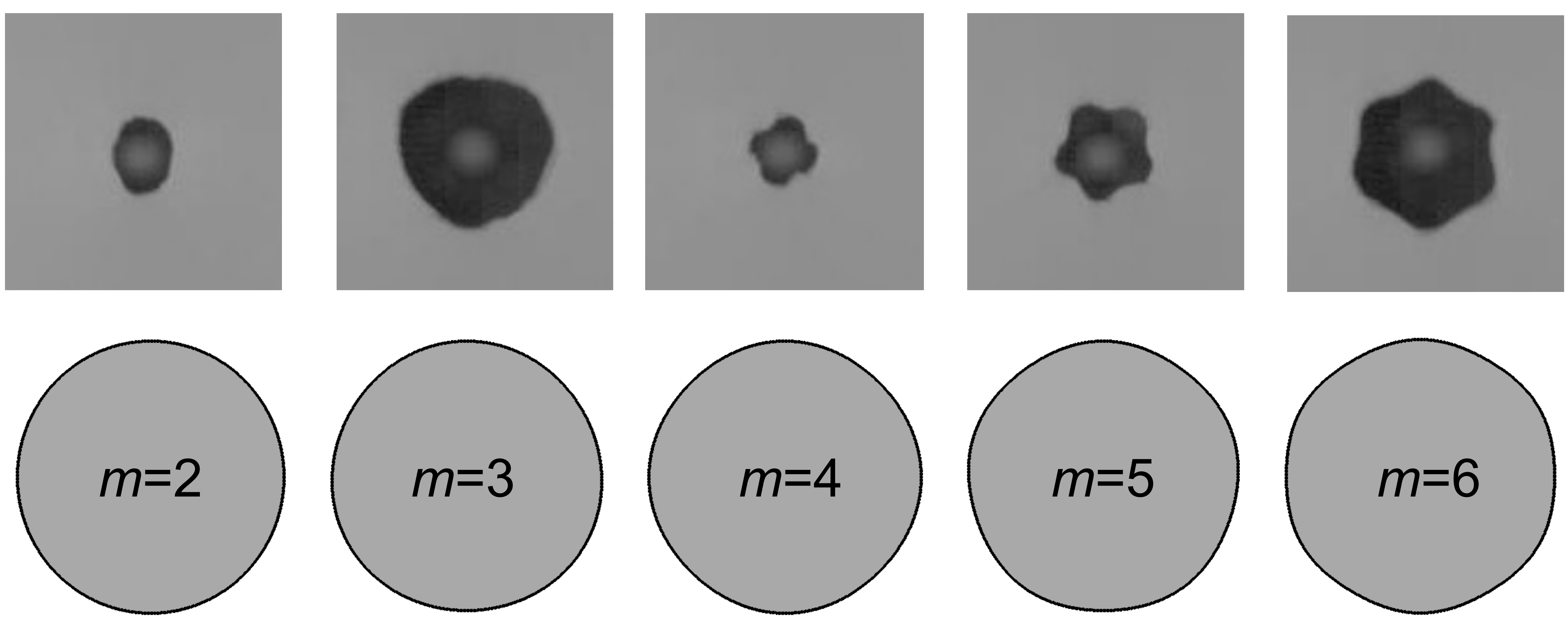
$$\frac{dU}{dt} = \frac{U^2}{\mathcal{R}}$$



This causes the reversal !

Experiments in linear regime

- Linear behavior: $a << R$
- Disks with 1% perturbation
- Water + powdered milk for visualization



A bit of theory:

Collapsing cavity with small azimuthal perturbation mode *m* around average $\bar{R}(t)$:

$$R(\theta, t) = \bar{R}(t) + a_m(t)\cos(m\theta)$$

corresponds to the flow potential:

$$\phi(r,t) = Q(t)\log r + d_m(t) r^{-m}\cos(m\theta)$$

The kinematic boundary condition at the cavity wall gives:

$$\left.\frac{\partial \phi}{\partial r}\right|_{r=\bar{R}(t)} = \frac{\partial R}{\partial t} \quad \Rightarrow \quad Q(t) = \bar{R}\dot{\bar{R}} \;; \quad d_m(t) = \frac{-\bar{R}^{m+1}}{m}\left[\dot{a}_m + a_m \frac{\dot{\bar{R}}}{\bar{R}}\right]$$

Combining with Bernoulli between R_∞ and cavity wall $R(t)$ (dynamic b.c.):

$$\rho\left[\frac{\partial \phi}{\partial t} + \tfrac{1}{2}|\vec{\nabla}\phi|^2\right]_{R_\infty}^{R(t)} = (P_\infty - P_0) + \cancel{\gamma\kappa}$$

after linearizing in a_m provides the amplitude equation:

$$\ddot{a}_m + \left(\frac{2\dot{\bar{R}}}{\bar{R}}\right)\dot{a}_m + \left(\frac{\ddot{\bar{R}}}{\bar{R}}(1-m)\right)a_m = 0$$

L.E. Schmidt, N.C. Keim, W.W. Zhang, and S.R. Nagel, *Nature Phys.* **5**, 343 (2009)

Inserting 2D Rayleigh scaling

Using the 2D Rayleigh scaling for the average radius $\bar{R}(t)$

$$\bar{R}(t) = C\sqrt{R_0 V}(t_c - t)^{1/2}$$

in the linear amplitude equation

$$\ddot{a}_m \ + \ \left(\frac{2\dot{\bar{R}}}{\bar{R}}\right)\dot{a}_m \ + \ \left(\frac{\ddot{\bar{R}}}{\bar{R}}(1-m)\right)a_m = 0$$

we find the solution:

$$a_m(t) = a_m(0)\ \cos\left(\tfrac{1}{2}\sqrt{m-1}\,\log(t_c - t) + \widetilde{\delta}\right)$$

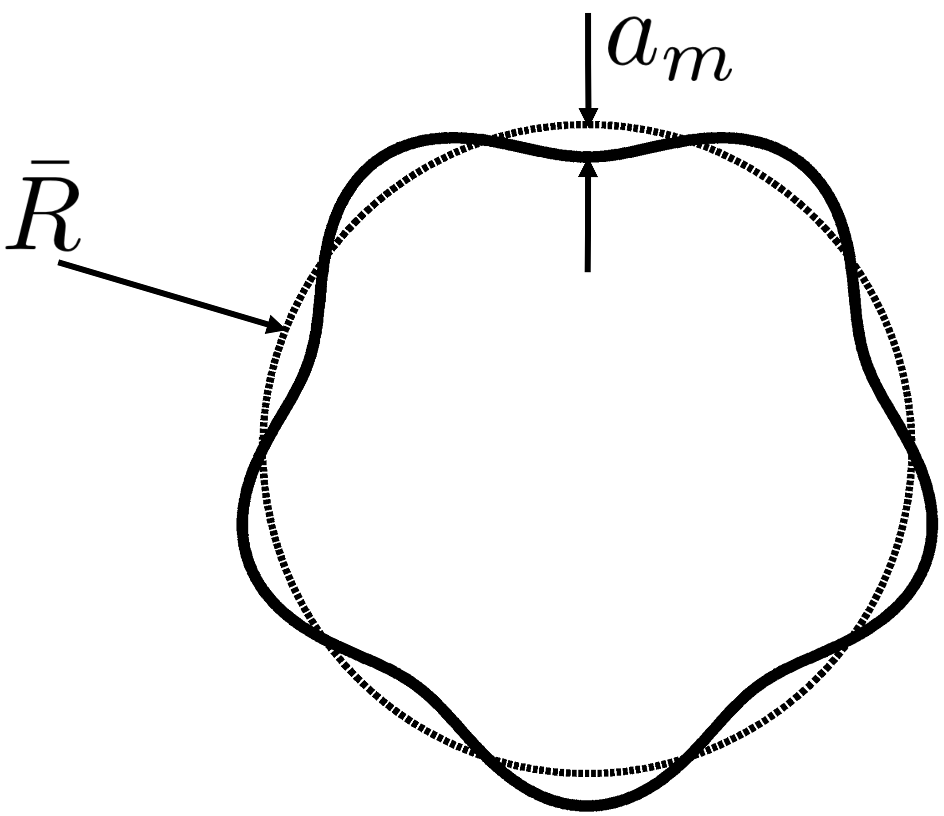
or:

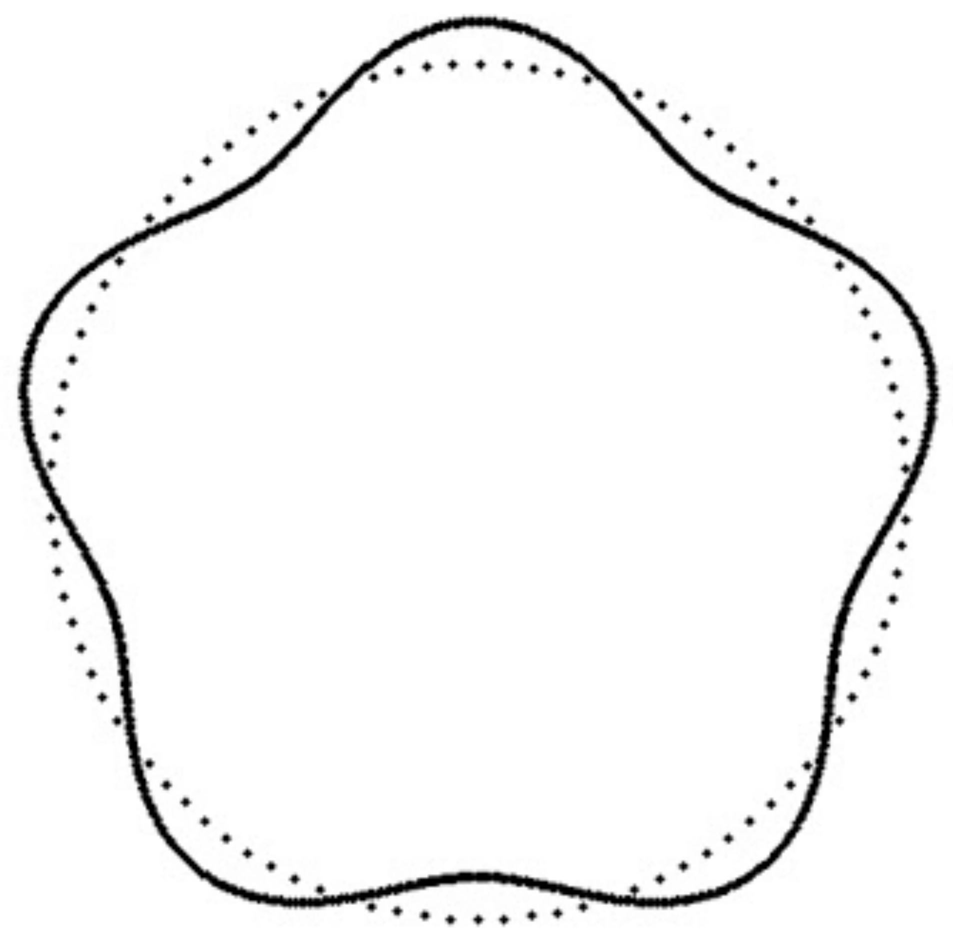
$$a_m(\bar{R}) = a_m(0)\ \cos\left(\sqrt{m-1}\,\log(\bar{R}/R_0) + \delta\right)$$

constant amplitude a_m ⇒
$a_m/\bar{R}$ diverges !

frequency chirps !

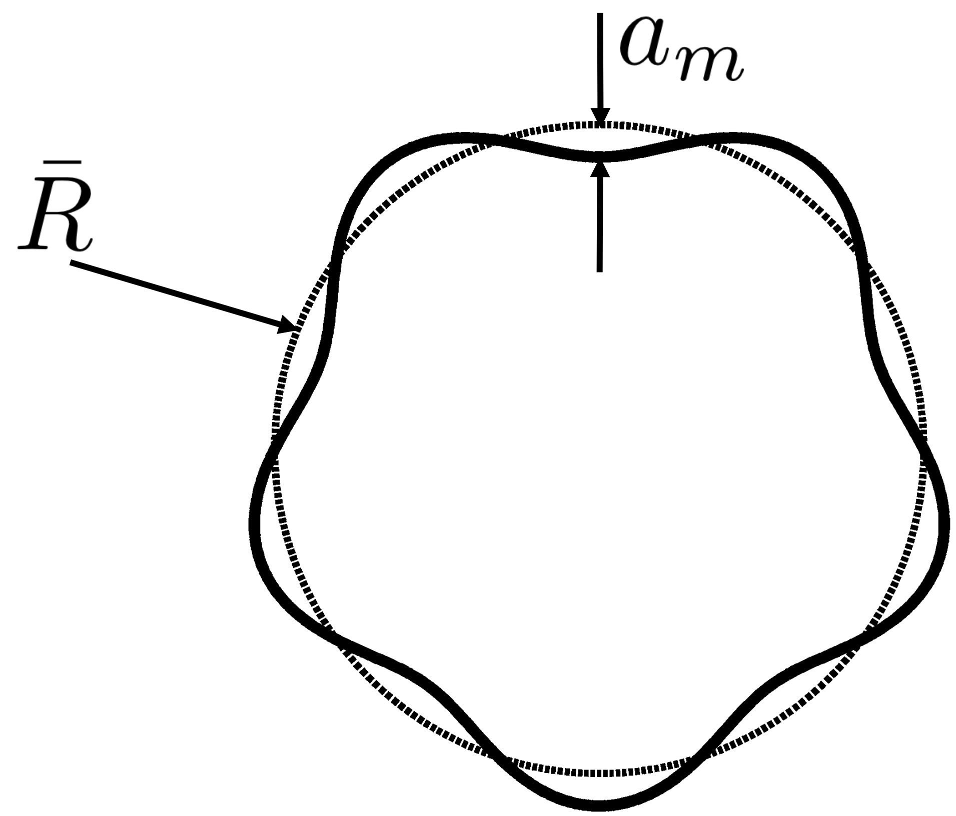
Results (linear regime)

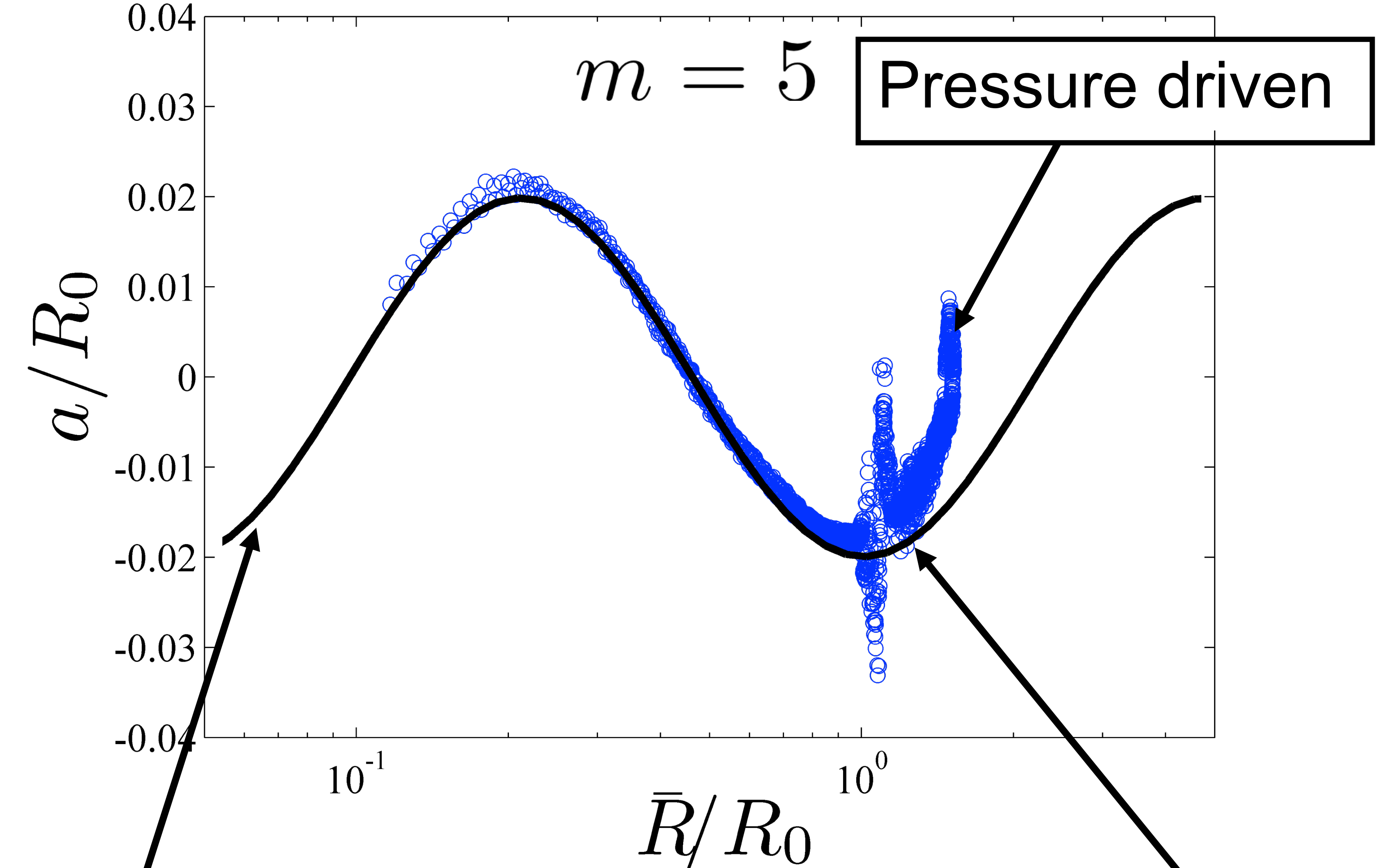




$$R(\theta, t) = \bar{R}(t) + a_m(t)\cos(m\theta)$$

Results (linear regime)

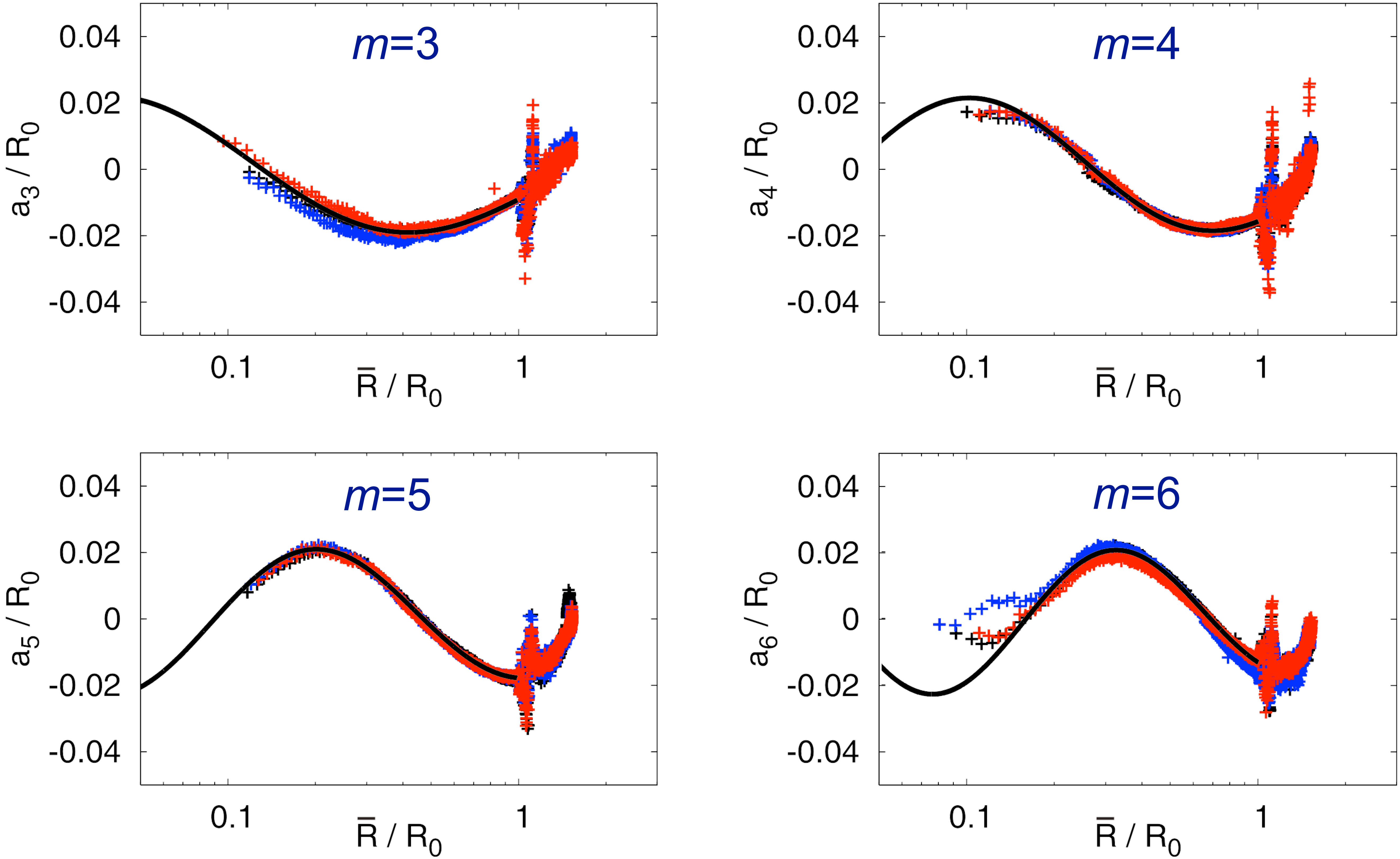




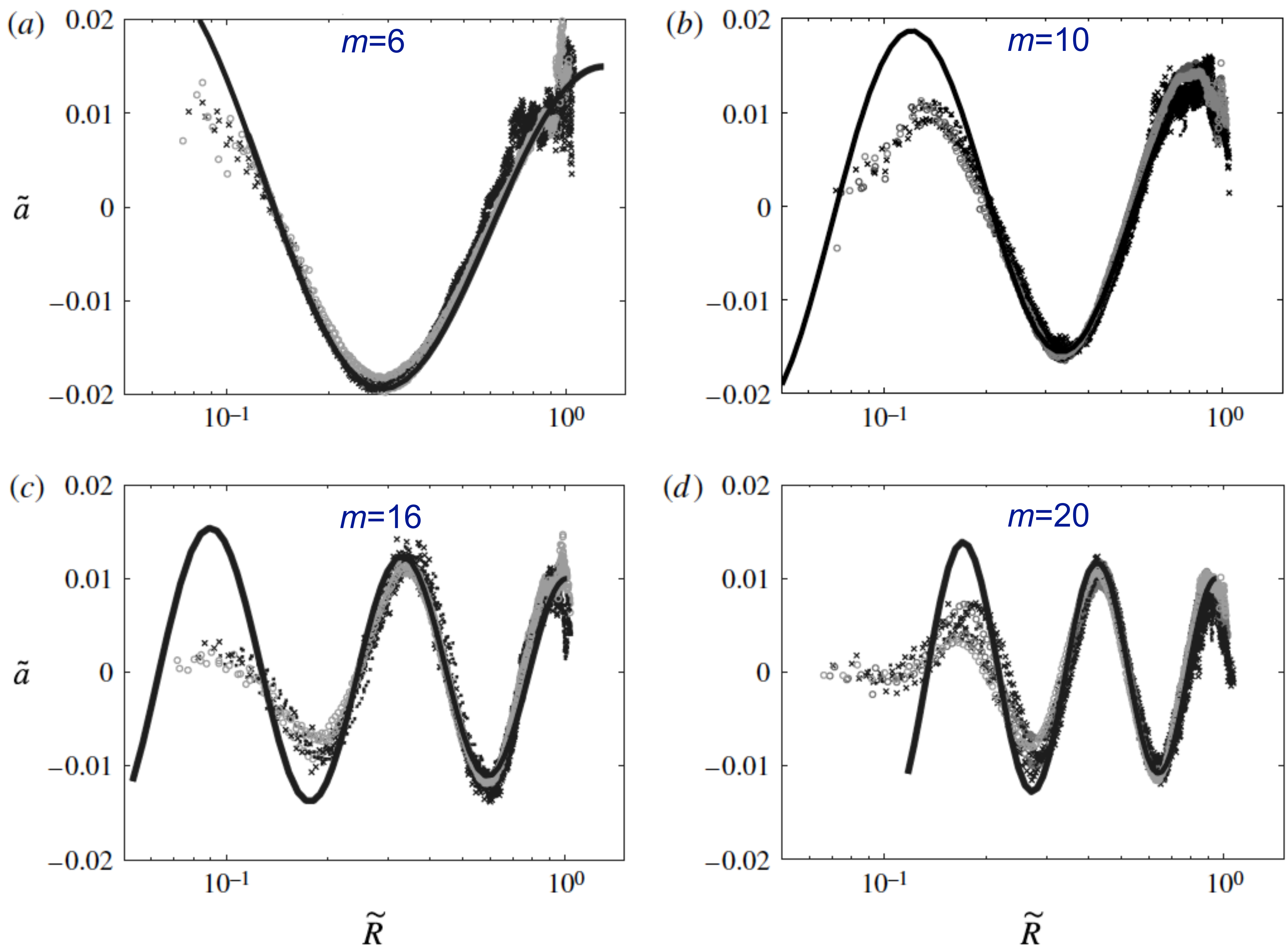
$$\boxed{a_m(\bar{R}) = a_m(0)\,\cos\left(\sqrt{m-1}\,\log(\bar{R}/R_0) + \delta\right)}$$

$$\boxed{\bar{R} \approx R_0}$$

Linear regime, 1% perturbation



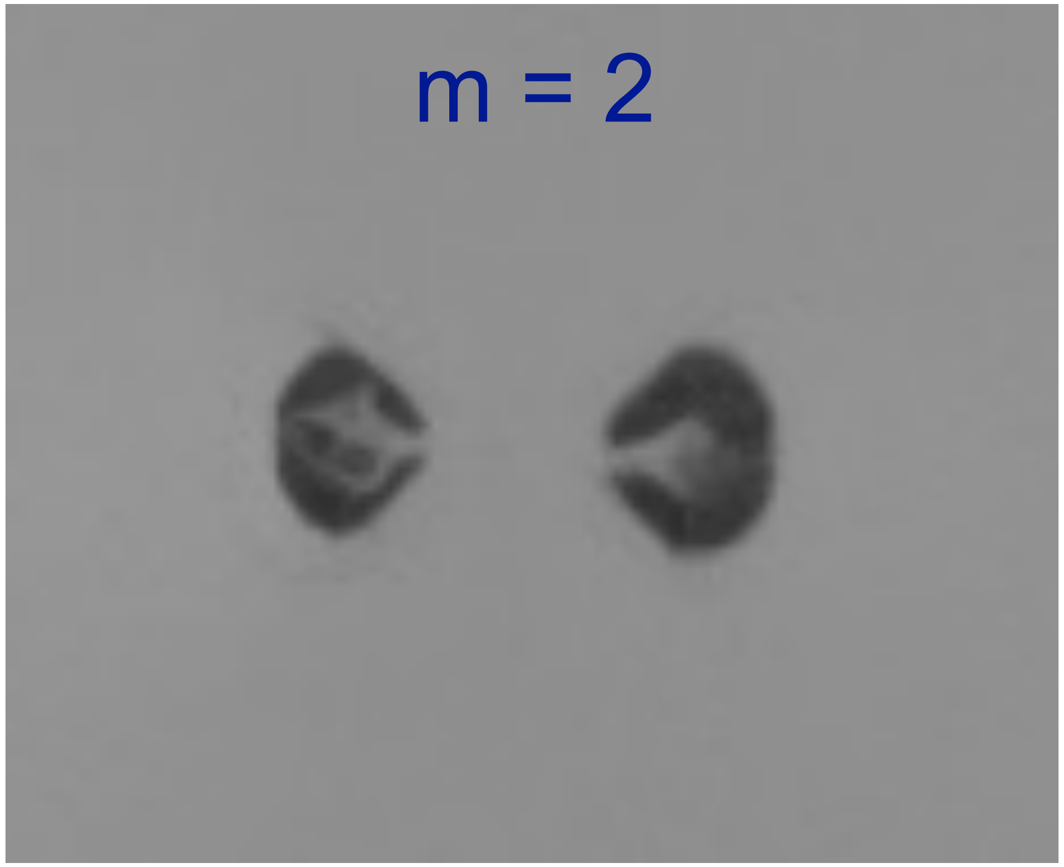
Linear regime, 1% perturbation

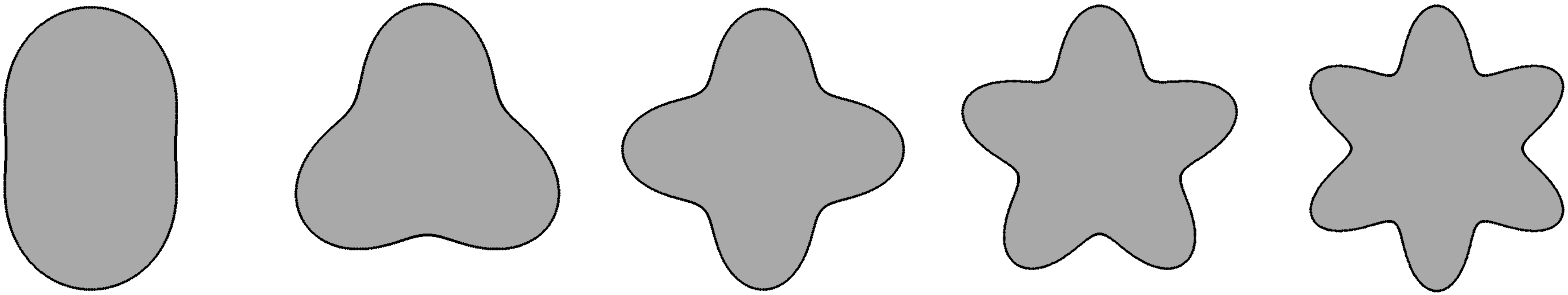


Experiments in the non-linear regime

(perturbations a comparable to disc radius R_0)

- jet formation

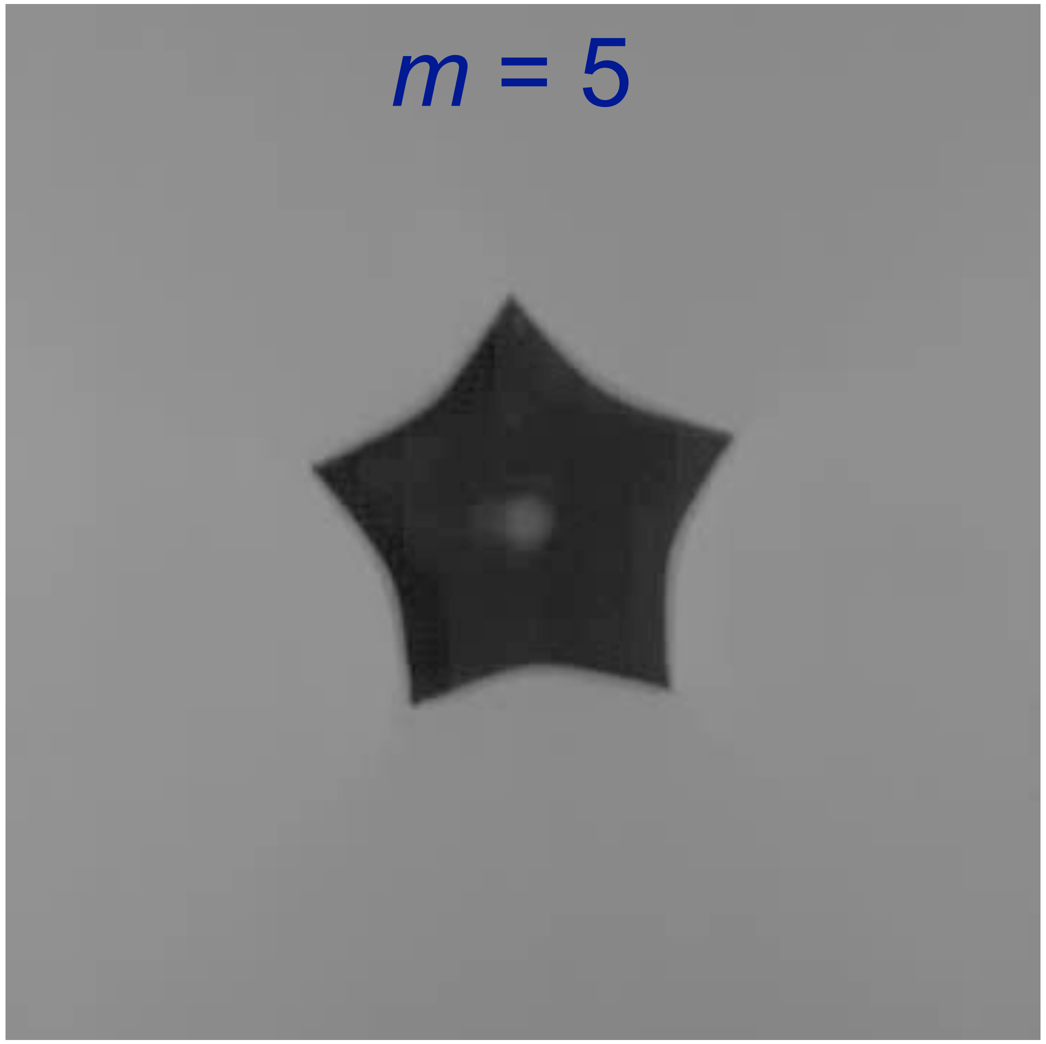


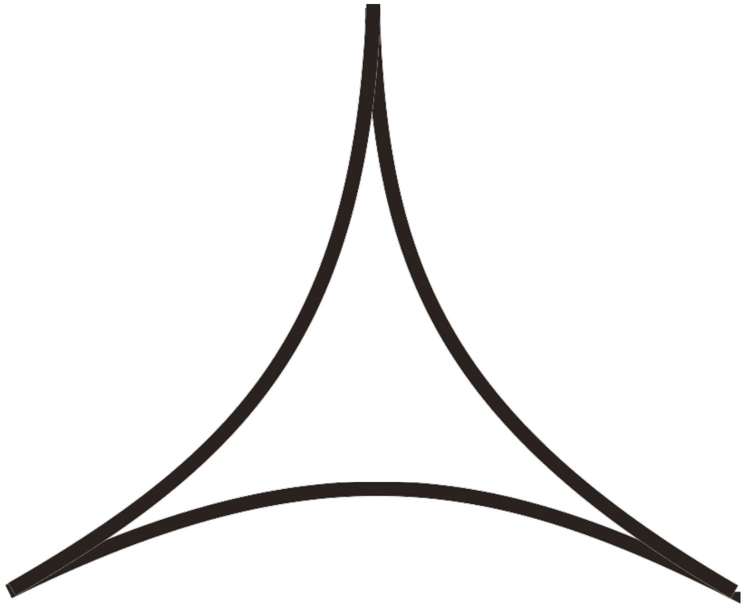


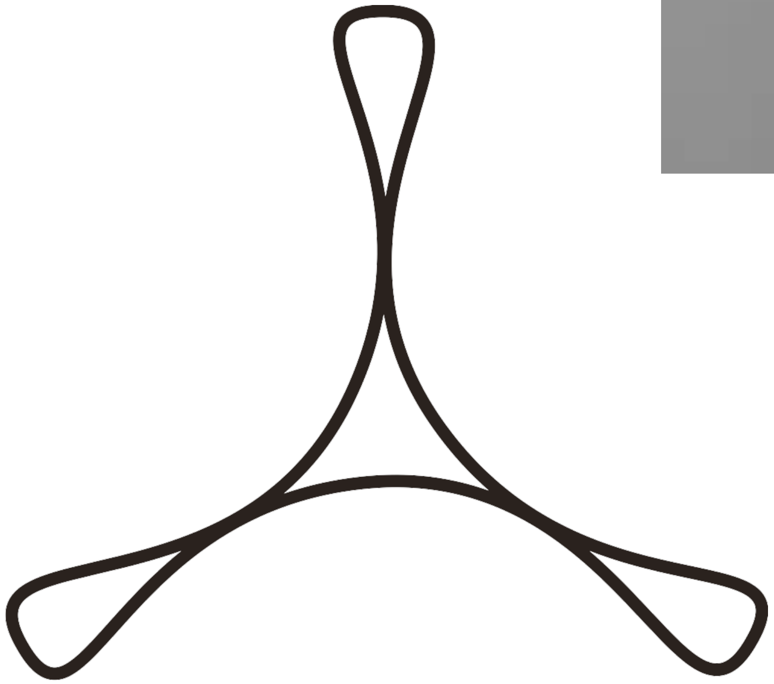
Experiments in the non-linear regime

(perturbations a comparable to disc radius R_0)

- jet formation
- cusp formation ($m \geq 3$)



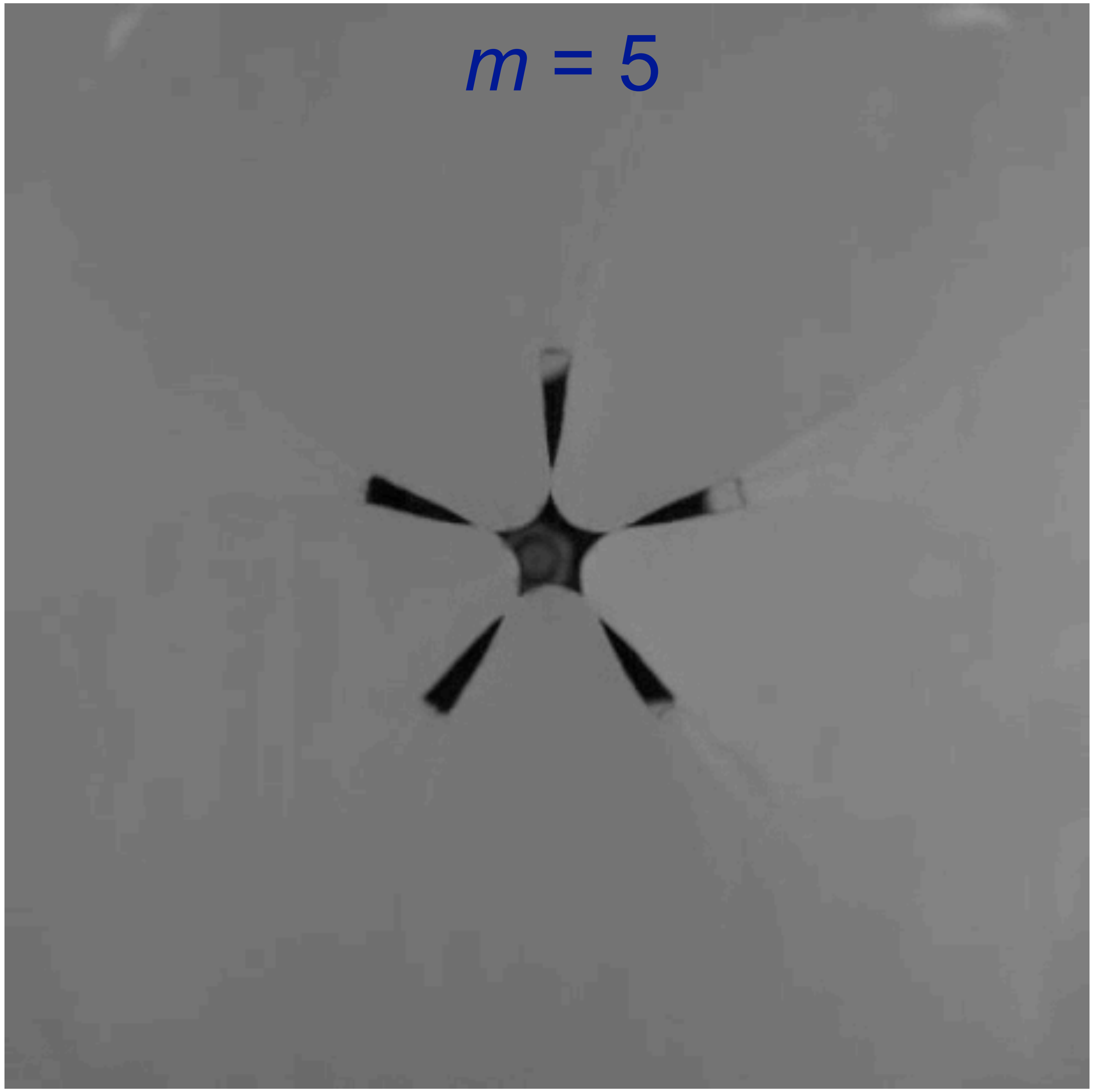




Experiments in the non-linear regime

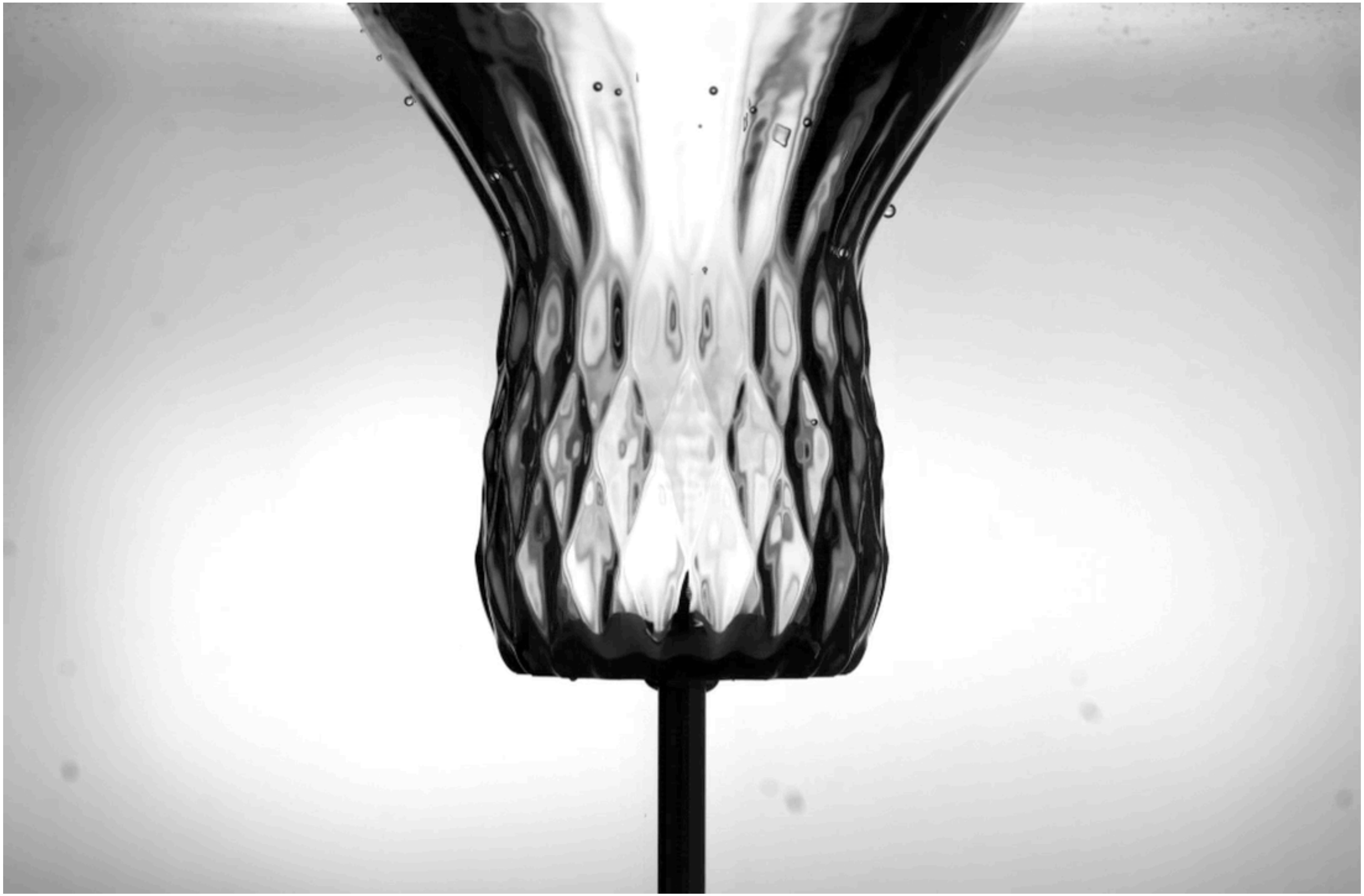
(perturbations a comparable to disc radius R_0)

- jet formation
- cusp formation ($m \geq 3$)
- subcavities

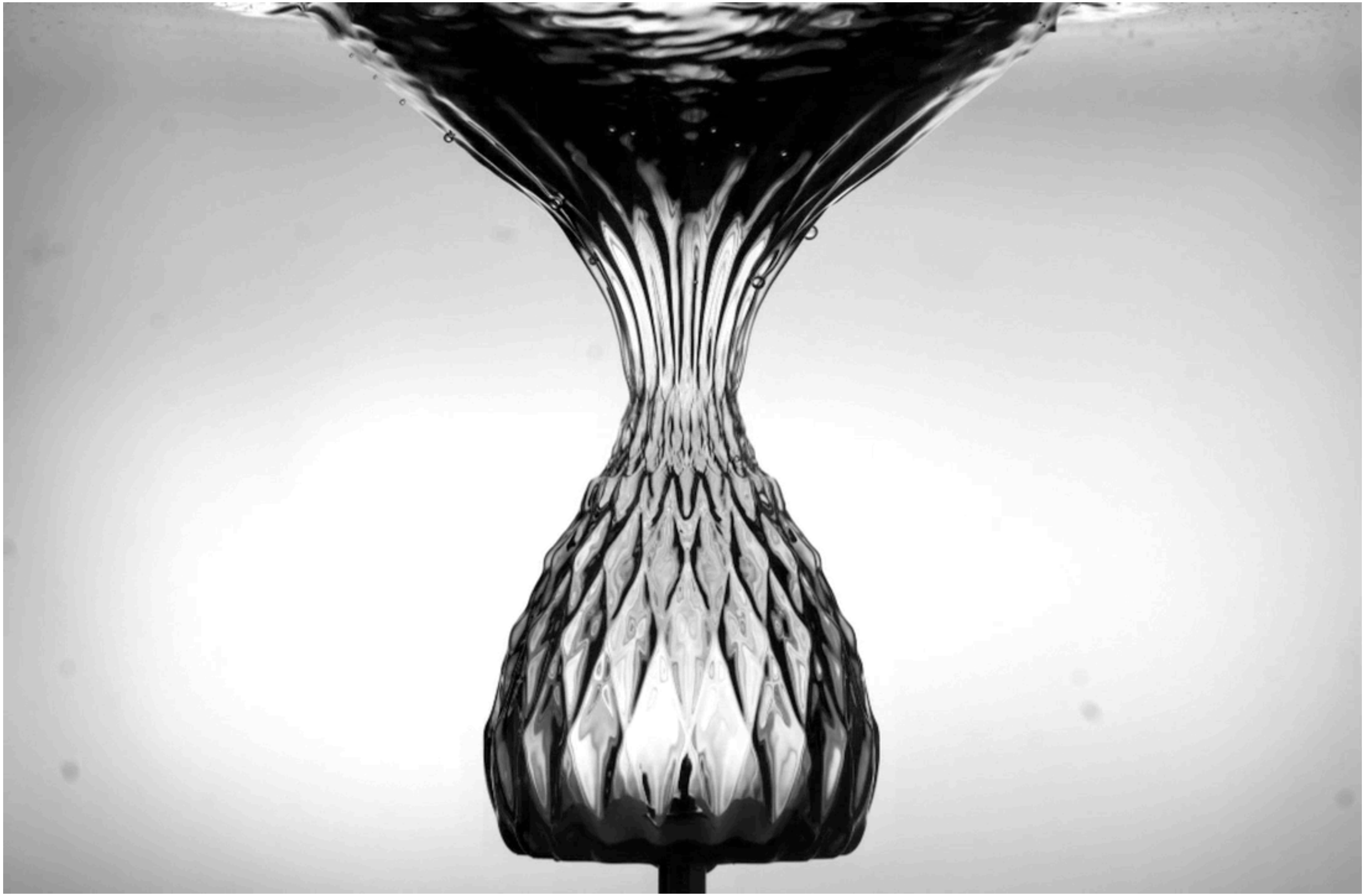


A side view (m=20, 10% perturbation)

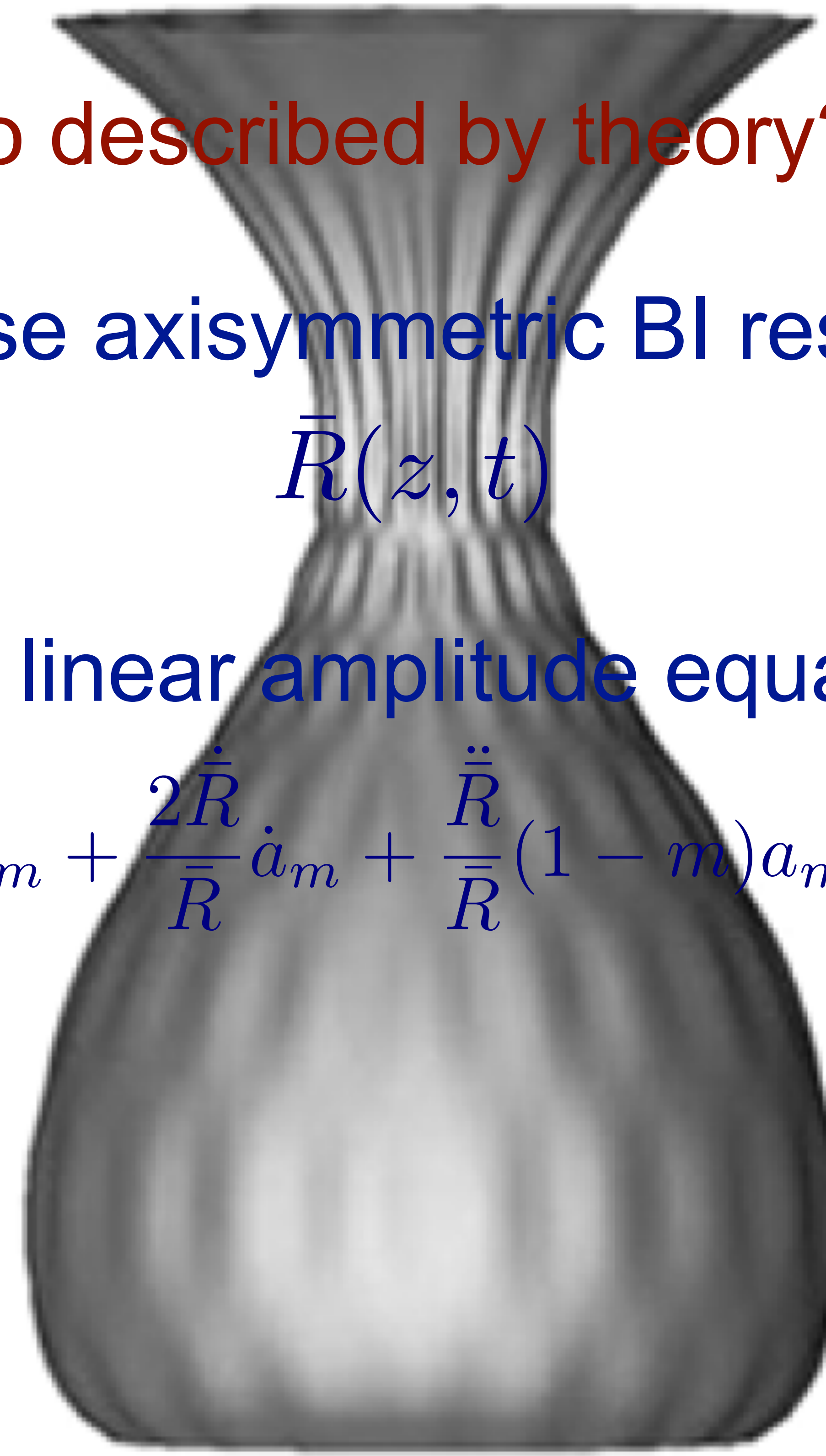
A side view (m=20, 10% perturbation)



A side view (m=20, 10% perturbation)



Understanding the side view pattern



Also described by theory?

- use axisymmetric BI result

$$\bar{R}(z,t)$$

- in linear amplitude equation

$$\ddot{a}_m + \frac{2\dot{\bar{R}}}{\bar{R}}\dot{a}_m + \frac{\ddot{\bar{R}}}{\bar{R}}(1-m)a_m = 0$$

O. Enríquez *et al.*, JFM **701**, 40-58 (2012).

Conclusions

Void creation and collapse:

- experiment & BI numerics agree wonderfully
- 2D Rayleigh equation captures essential dynamics
- Airflow becomes supersonic in the neck

Breaking axisymmetry:

- perfect agreement small oscillations with Schmidt’s theory
- large perturbations show cusp, jet & subcavity formation
- axisymmetric BI result + Schmidt’s theory captures pineapple shape cavity


THANK YOU !