Simulations of a two-fluid model, sharpening the interface

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Arris Tijsseling
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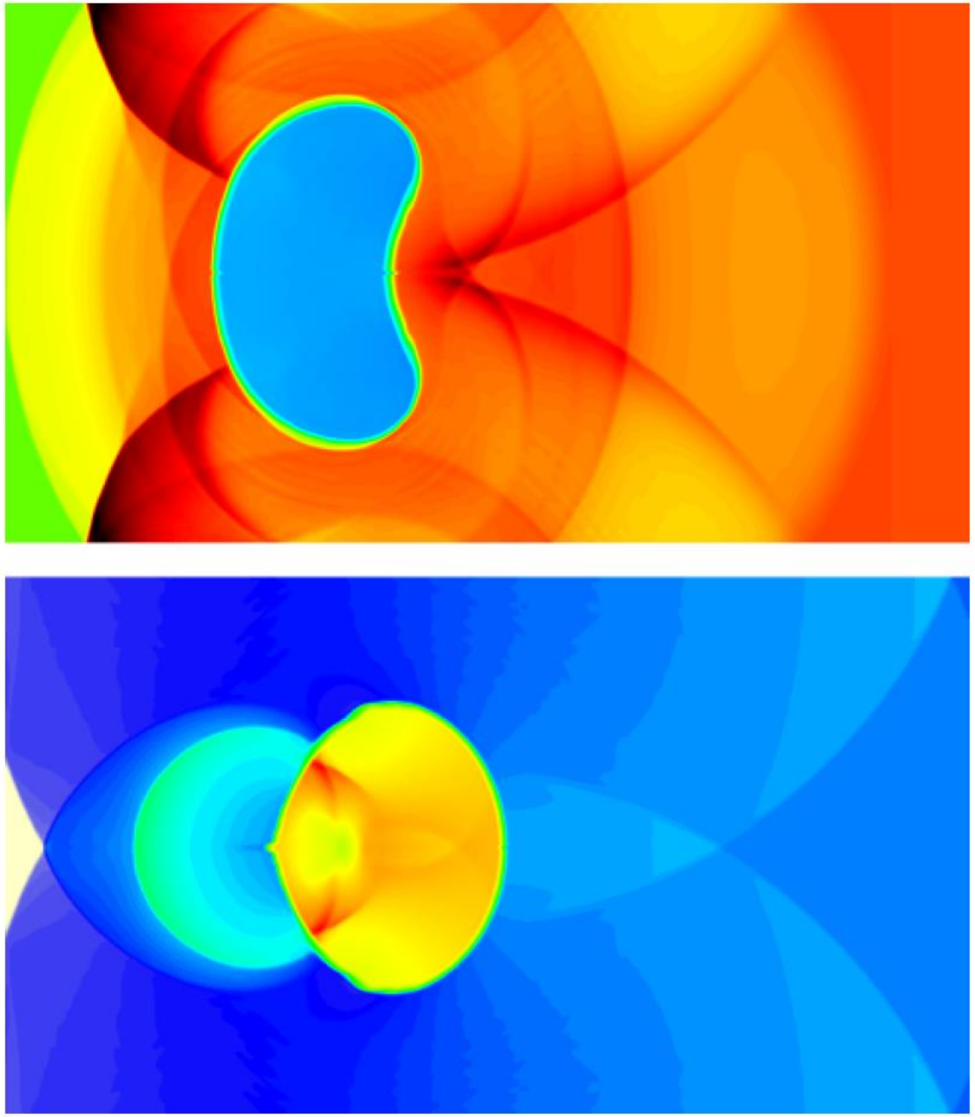
Sloshing of Liquefied Natural Gas

Two-fluid models



J. Kreeft, B. Koren, A new formulation of Kapila's five-equation model for compressible two-fluid flow, and its numerical treatment, Journal of computational physics 229-18, 6220-6242 (2010)

Context

Gas and liquid compressibility

Density ratio

Phase transition

Context

Gas and liquid compressibility

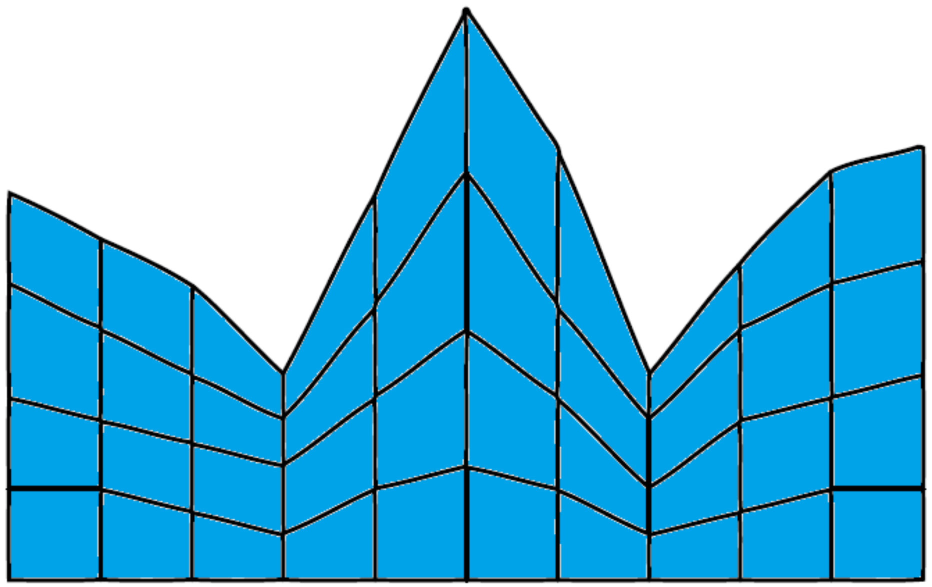
Density ratio

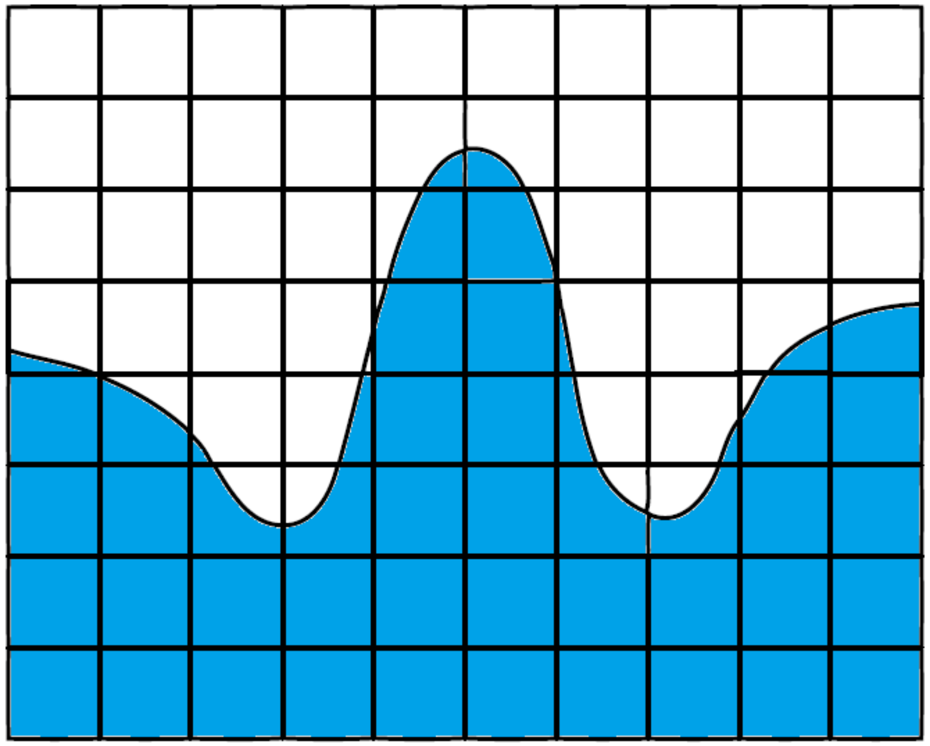
Phase transition

But not

- Surface tension
- Viscosity
- Heat exchange
- Mixing

Two-fluid interfaces





Contents

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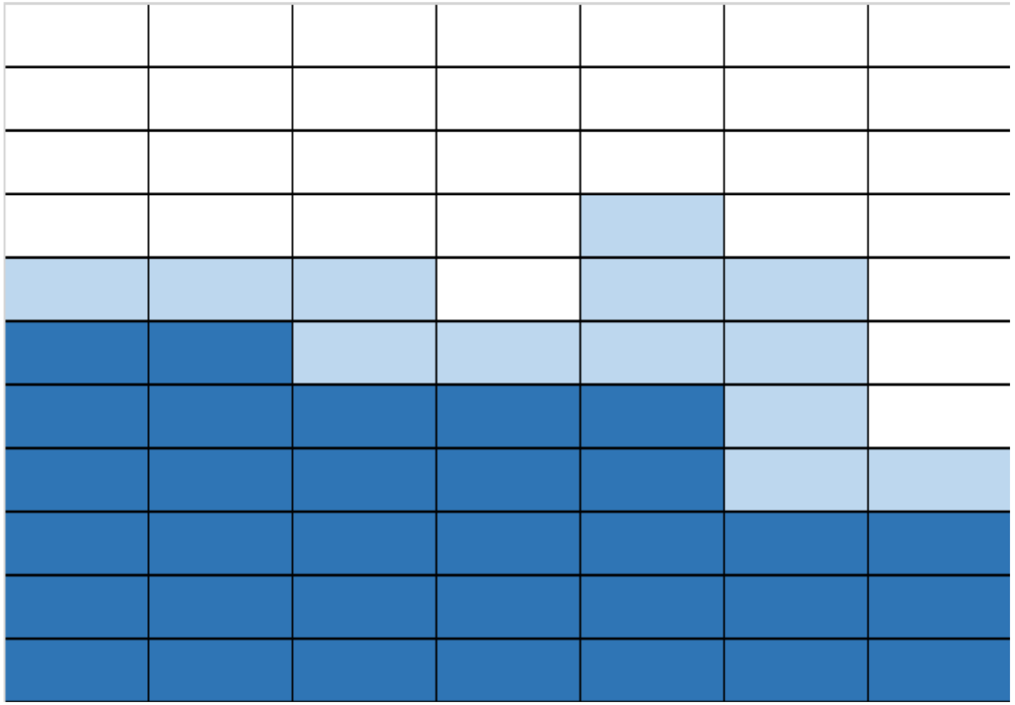
Numerical method

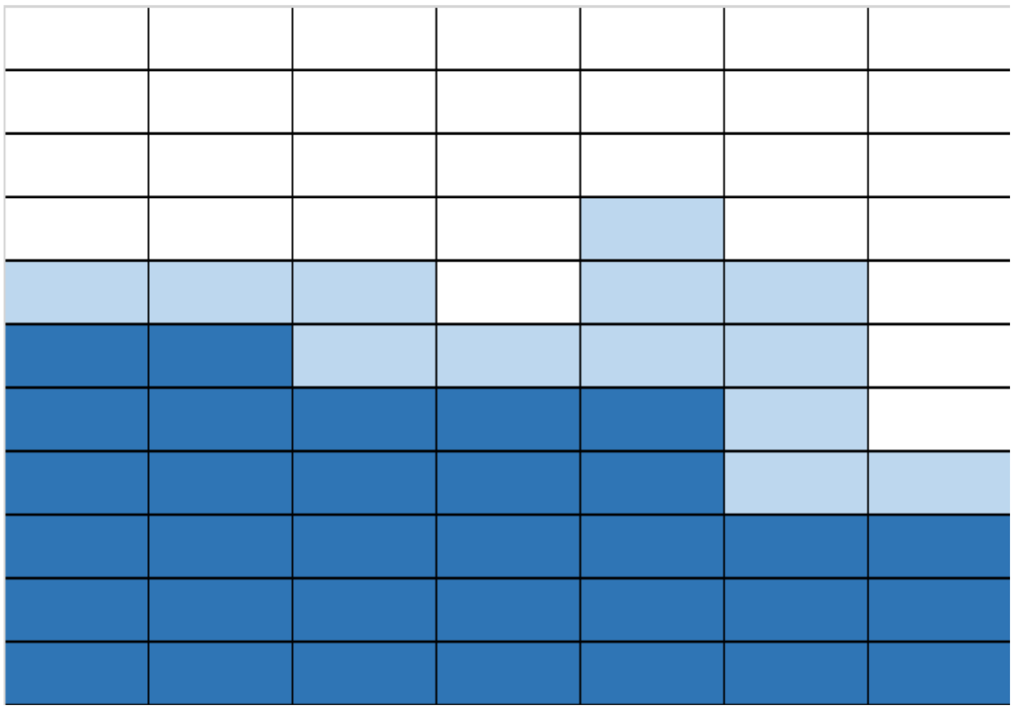
- Spatial reconstruction
- Limiters

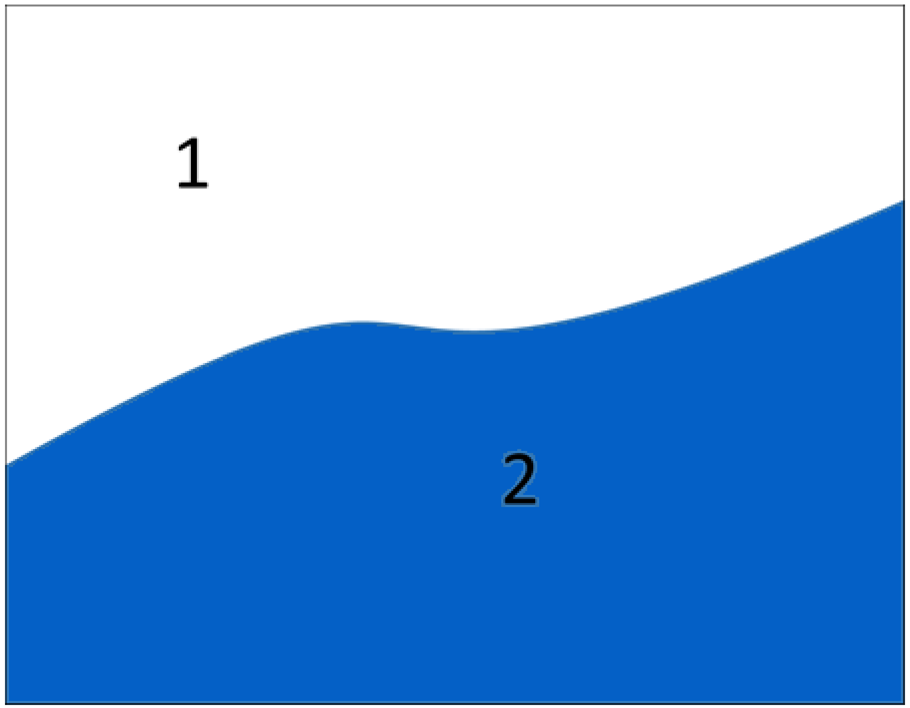
Conclusions and future work

The two-fluid model



The two-fluid model





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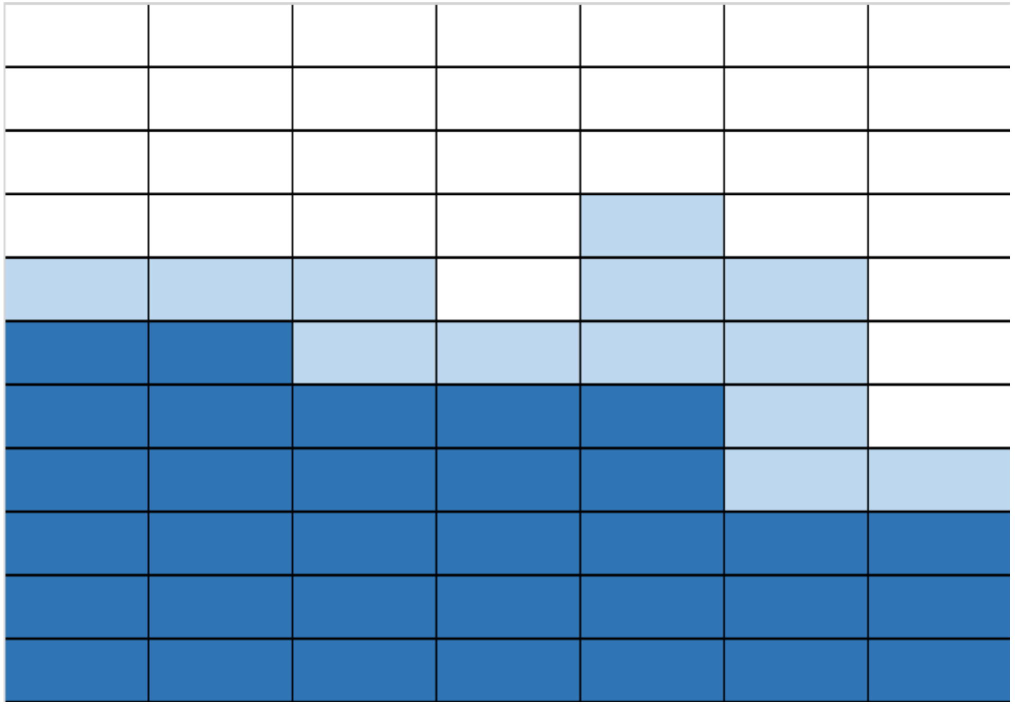
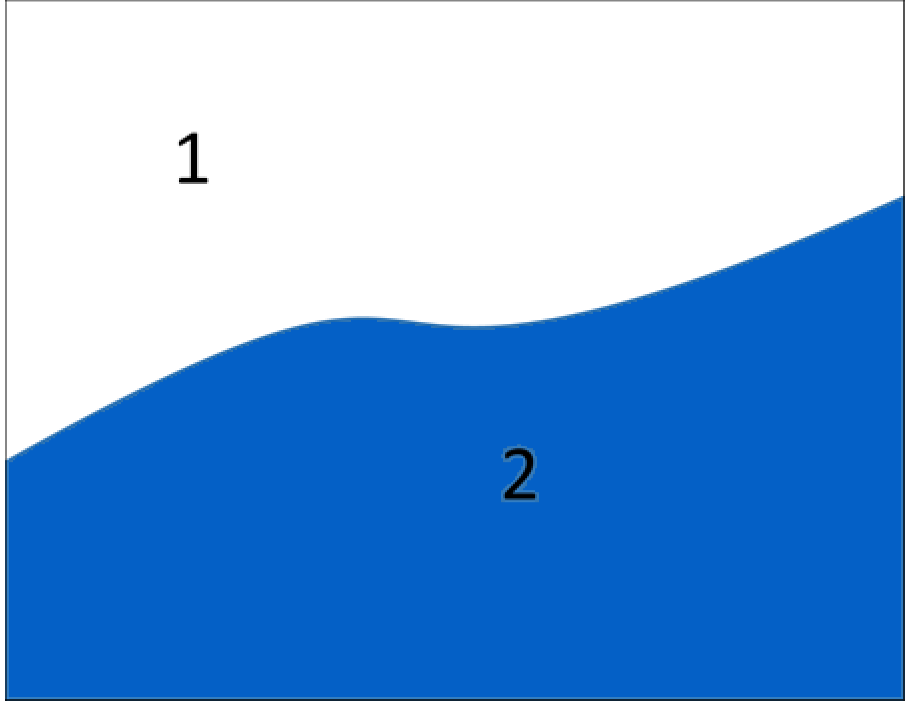
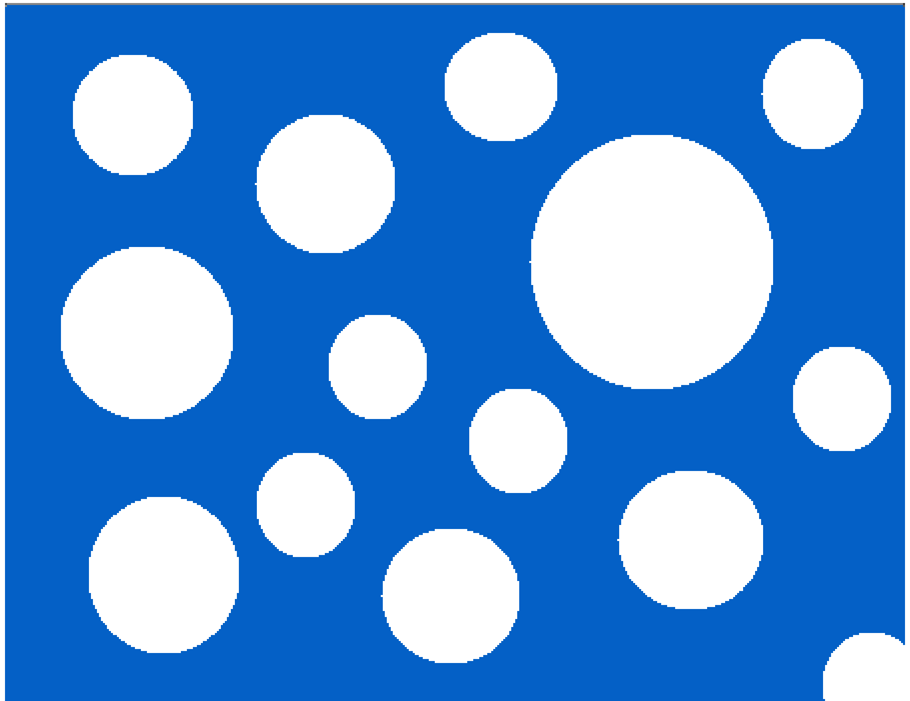




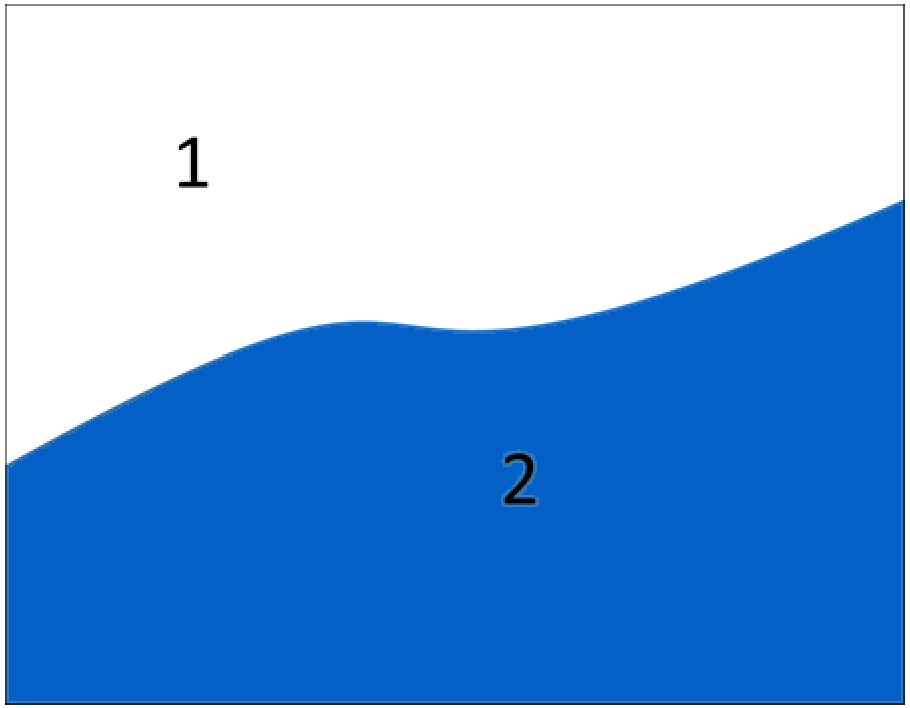


The two-fluid model

Bulk quantities:

$$\rho, u, v, p, e, E$$

Volume fraction α



The two-fluid model

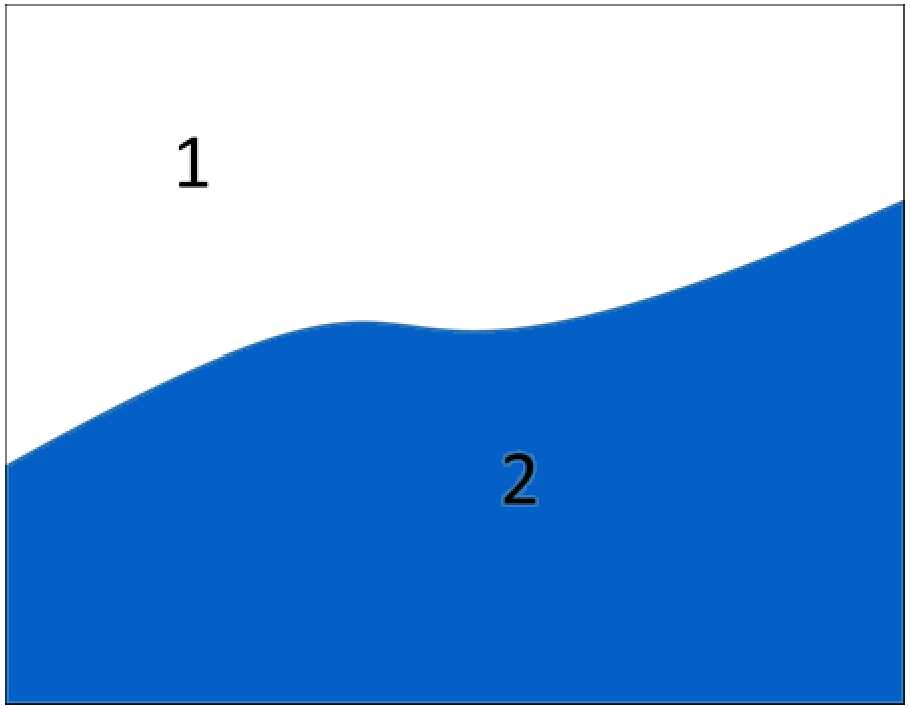
Bulk quantities:

$$\rho, u, v, p, e, E$$

Volume fraction α

Individual quantities:

$$\begin{aligned}
\rho &= \alpha\rho_1 + (1-\alpha)\rho_2, \\
u &= u_1 = u_2, \\
v &= v_1 = v_2 \\
p &= p_1 = p_2, \\
\rho e &= \alpha\rho_1 e_1 + (1-\alpha)\rho_2 e_2, \\
E &= e + \frac{1}{2}(u^2 + v^2)
\end{aligned}$$



The two-fluid model

Bulk mass conservation

$$\rho_t + (\rho u)_x + (\rho v)_y = 0,$$

Bulk momentum conservation

$$(\rho u)_t + (\rho u^2 + p)_x + (\rho uv)_y = 0,$$

$$(\rho v)_t + (\rho uv)_x + (\rho v^2 + p)_y = 0,$$

Bulk energy conservation

$$(\rho E)_t + (u(\rho E + p))_x + (v(\rho E + p))_y = 0,$$

The two-fluid model

Bulk mass conservation $\rho_t + (\rho u)_x + (\rho v)_y = 0,$

Bulk momentum conservation $(\rho u)_t + (\rho u^2 + p)_x + (\rho uv)_y = 0,$

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Bulk energy conservation $(\rho E)_t + (u(\rho E + p))_x + (v(\rho E + p))_y = 0,$

Mass conservation fluid 1 $(\alpha\rho_1)_t + (\alpha\rho_1 u)_x + (\alpha\rho_1 v)_y = 0,$

Advection of volume fraction $(\alpha)_t + (\alpha u)_x + (\alpha v)_y = (\alpha - \varphi)(u_x + v_y),$

A. Kapila, R. Menikoff, J. Bdzil, S. Son, Two-phase modeling of deflagration-to-detonation transition in granular materials: Reduced equations, Physics of Fluids,13, 3002, (2001)

The two-fluid model

Bulk mass conservation $$\rho_t + (\rho u)_x + (\rho v)_y = 0,$$

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Advection of volume fraction $$(\alpha)_t + (\alpha u)_x + (\alpha v)_y = (\alpha - \varphi)(u_x + v_y),$$

Parameter φ $$\varphi = \alpha(1-\alpha)\frac{\rho_1 c_1^2 - \rho_2 c_2^2}{(1-\alpha)\rho_1 c_1^2 + \alpha\rho_2 c_2^2},$$

Equations of state $$e_1 = f(\rho_1, p), e_2 = f(\rho_2, p),$$

A. Kapila, R. Menikoff, J. Bdzil, S. Son, Two-phase modeling of deflagration-to-detonation transition in granular materials: Reduced equations, Physics of Fluids,13, 3002, (2001)

The two-fluid model

Hyperbolic System:

Characterics propagate with velocities
$u - c, u, u + c,$
$v - c, v, v + c,$

c: mixture speed of sound:

$$\frac{1}{\rho c^2} = \frac{\alpha}{\rho_1 c_1^2} + \frac{1-\alpha}{\rho_2 c_2^2}$$

$$\rho_t + (\rho u)_x + (\rho v)_y = 0,$$

$$(\rho u)_t + (\rho u^2 + p)_x + (\rho u v)_y = 0,$$

$$(\rho v)_t + (\rho u v)_x + (\rho v^2 + p)_y = 0,$$

$$(\rho E)_t + (u(\rho E + p))_x + (v(\rho E + p))_y = 0,$$

$$(\alpha \rho_1)_t + (\alpha \rho_1 u)_x + (\alpha \rho_1 v)_y = 0,$$

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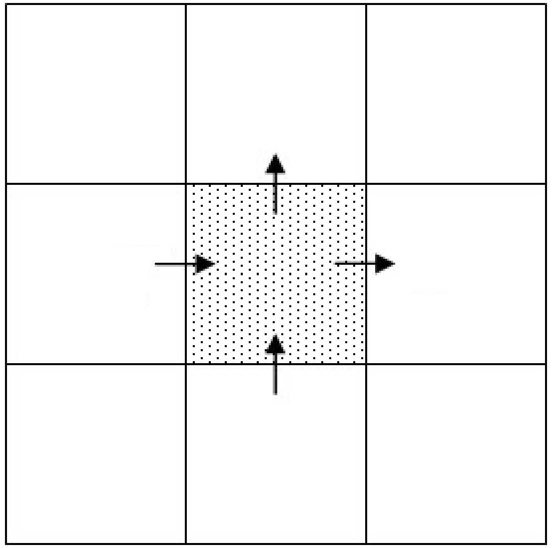
Numerical method

Finite volume discretization



Basic explicit time step

$$Q_{i,j}^{n+\frac{1}{2}} = Q_{i,j}^{n} + \frac{\Delta t}{\Delta x}\left(f_{i-\frac{1}{2},j}^{n} - f_{i+\frac{1}{2},j}^{n}\right) + \Delta t S_{x,i,j}^{n}$$

$$Q_{i,j}^{n+1} = Q_{i,j}^{n+\frac{1}{2}} + \frac{\Delta t}{\Delta y}\left(g_{i,j-\frac{1}{2}}^{n} - g_{i,j+\frac{1}{2}}^{n}\right) + \Delta t S_{y,i,j}^{n}$$

$$\rho_t + (\rho u)_x + (\rho v)_y = 0,$$

$$(\rho u)_t + (\rho u^2 + p)_x + (\rho u v)_y = 0,$$

$$(\rho v)_t + (\rho u v)_x + (\rho v^2 + p)_y = 0,$$

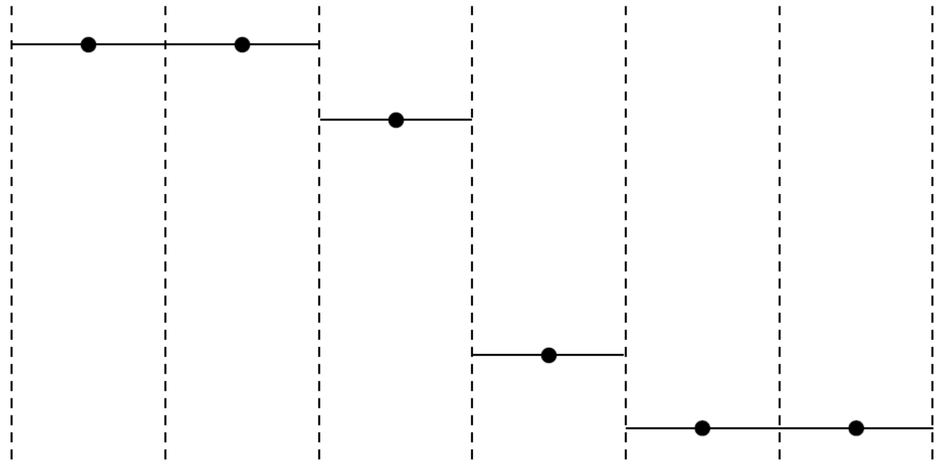
$$(\rho E)_t + (u(\rho E + p))_x + (v(\rho E + p))_y = 0,$$

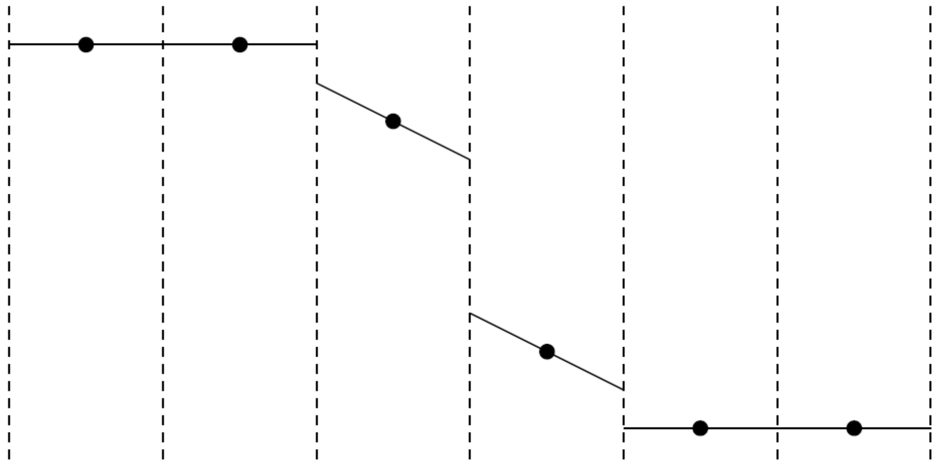
$$(\alpha \rho_1)_t + (\alpha \rho_1 u)_x + (\alpha \rho_1 v)_y = 0,$$

$$(\alpha)_t + (\alpha u)_x + (\alpha v)_y = (\alpha - \varphi)(u_x + v_y),$$

$$\boldsymbol{Q}_t + \boldsymbol{F}(\boldsymbol{Q})_x + \boldsymbol{G}(\boldsymbol{Q})_y = \boldsymbol{S},$$

Spatial reconstruction





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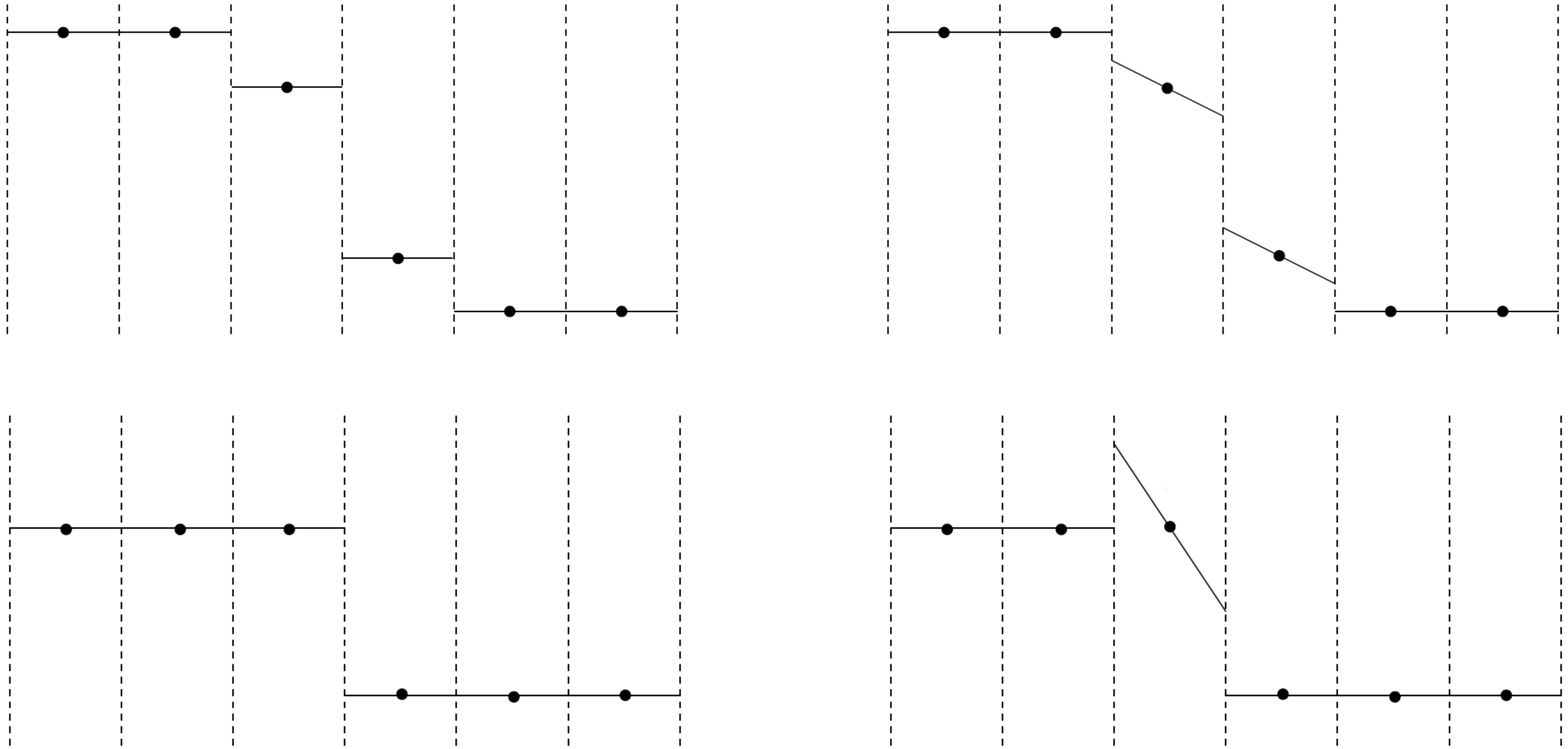
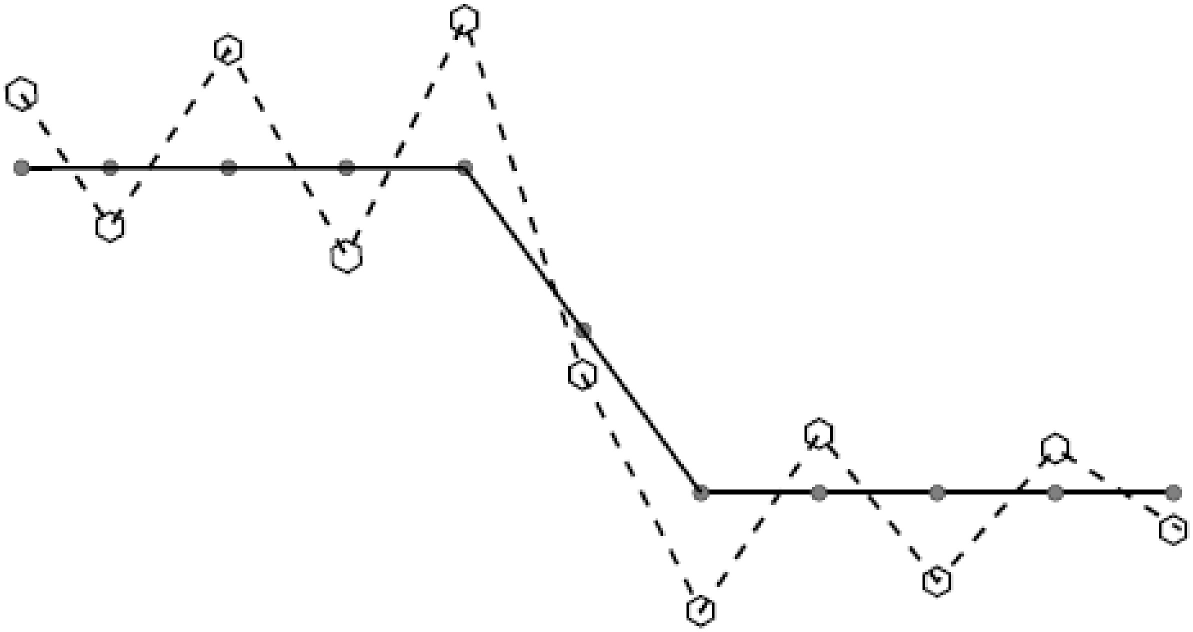


Spatial reconstruction

TVD schemes

$$\mathrm{TV} = \sum_i |q_{i+1} - q_i|$$



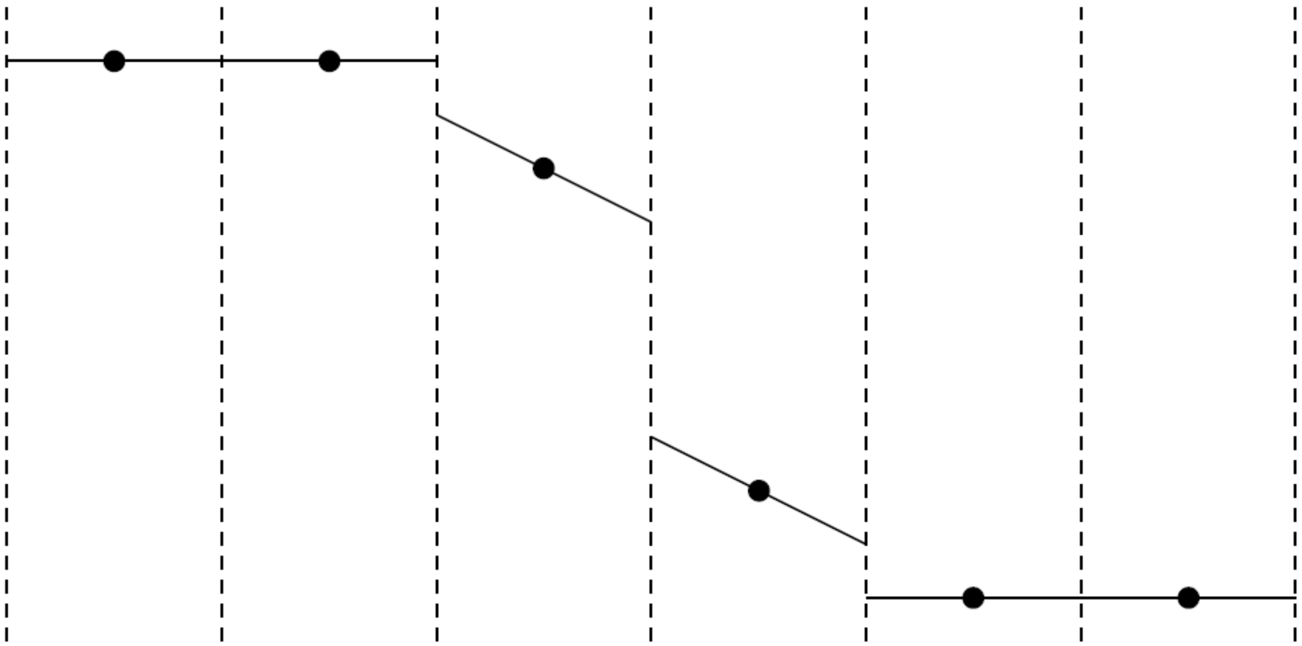
S. Godunov, A finite difference method for the computation of discontinuous solutions of the equations of fluid dynamics, Matematicheskii Sbornik, 47, 537-393, (1959)

Limiters

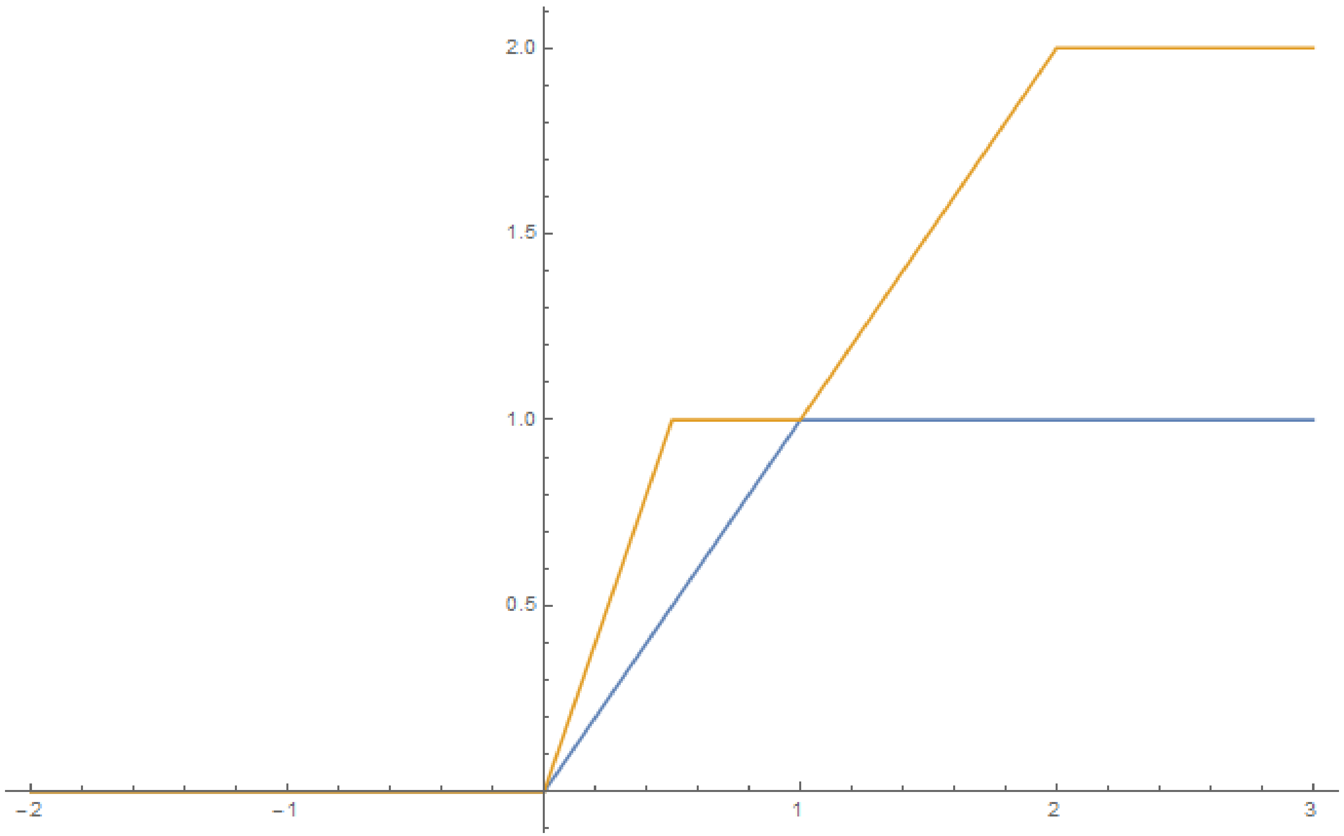
Slope is a function of differences

$$W^L_{i+\frac{1}{2}} = W_i + \frac{1}{2}\psi(r)\,(W_{i+1} - W_i)$$

$$r = \frac{W_{i+1} - W_i}{W_i - W_{i-1}}$$

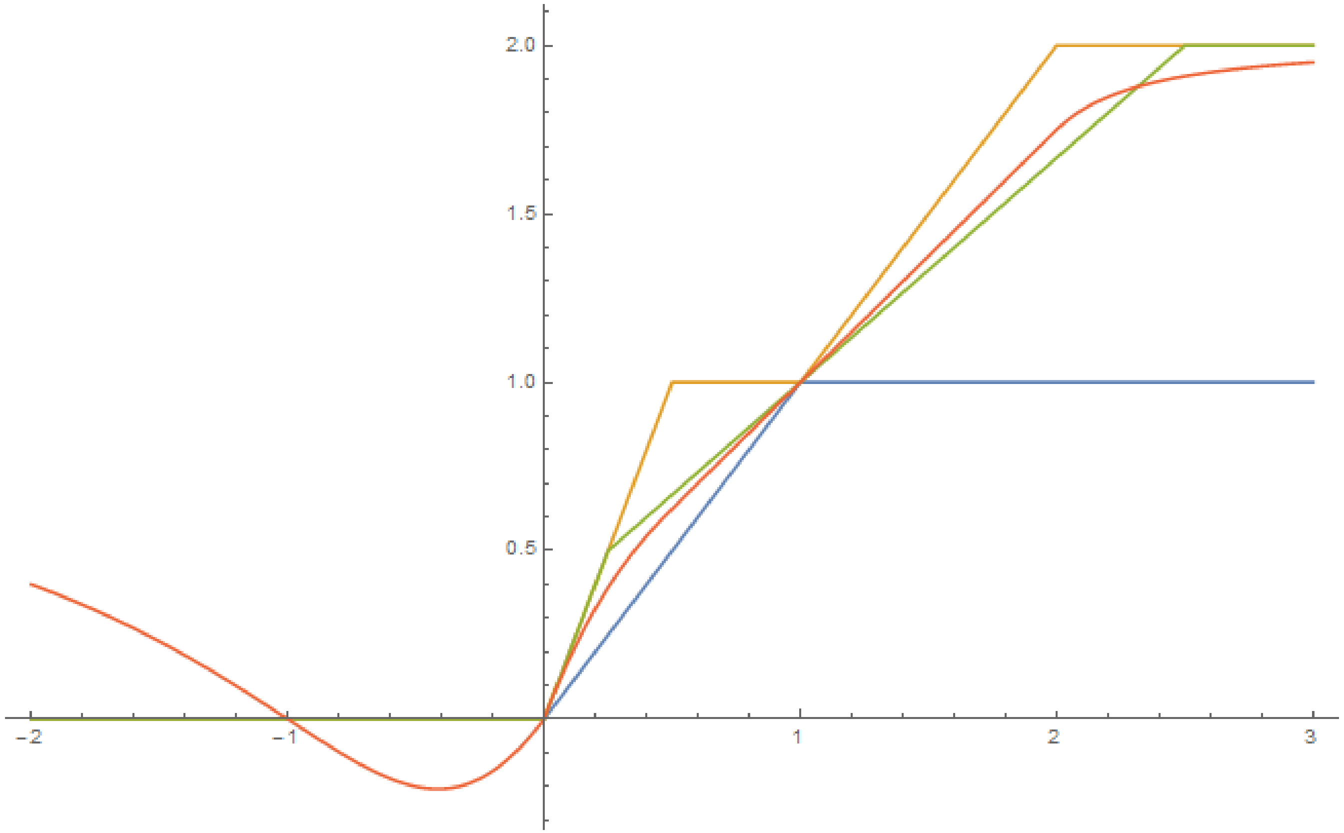


Limiters



Superbee, Minmod

Limiters

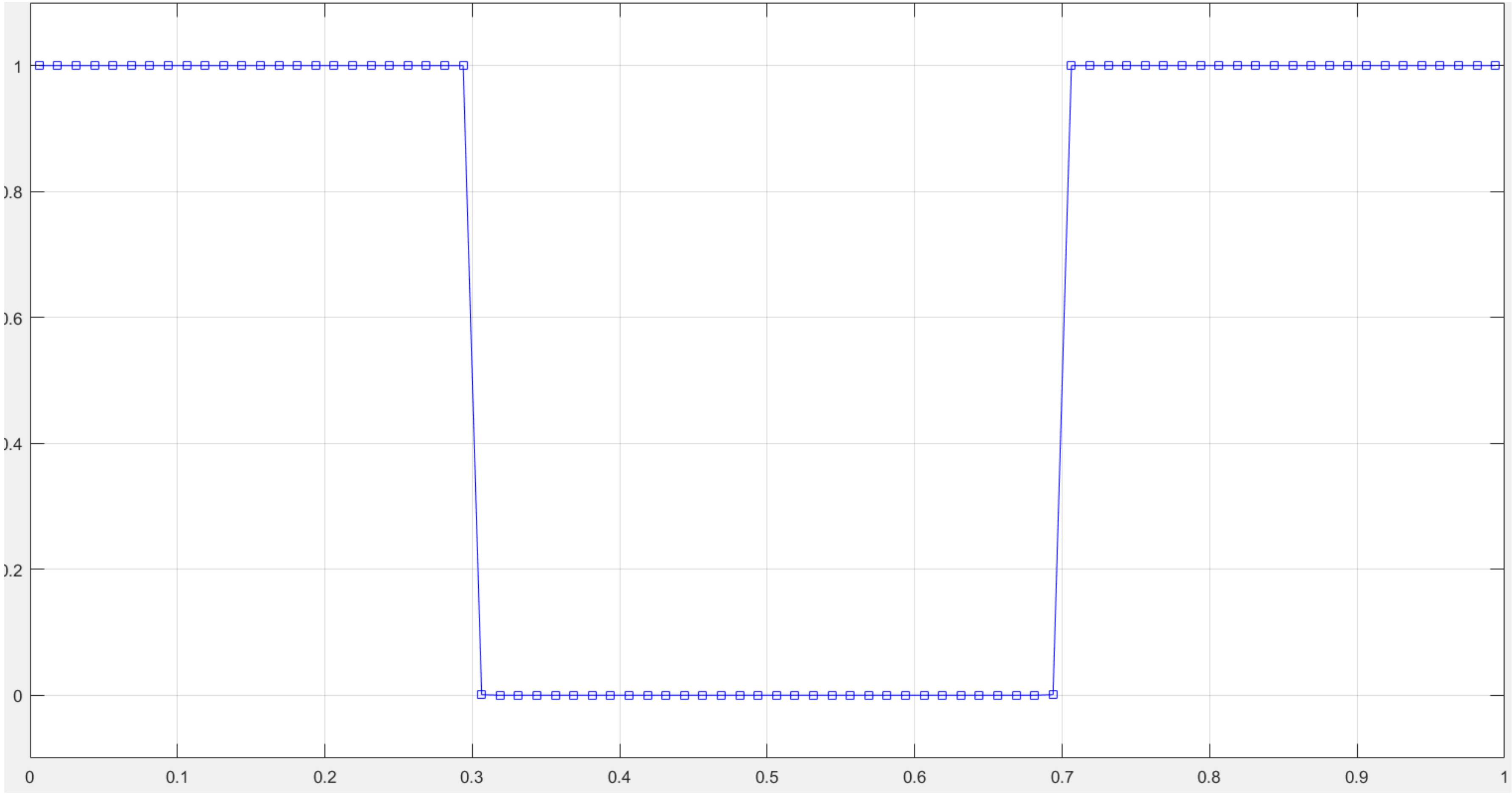


Superbee, Minmod, Koren, TCDF

D. Zhang, C. Jiang, D. Liang, L. Cheng, A review on TVD schemes and a refined flux-limiter for steady-state computations, Journal of computational physics 302-1, 114-154 (2015)

Comparison of limiters

- **1D Linear advection equation:**



Comparison of limiters

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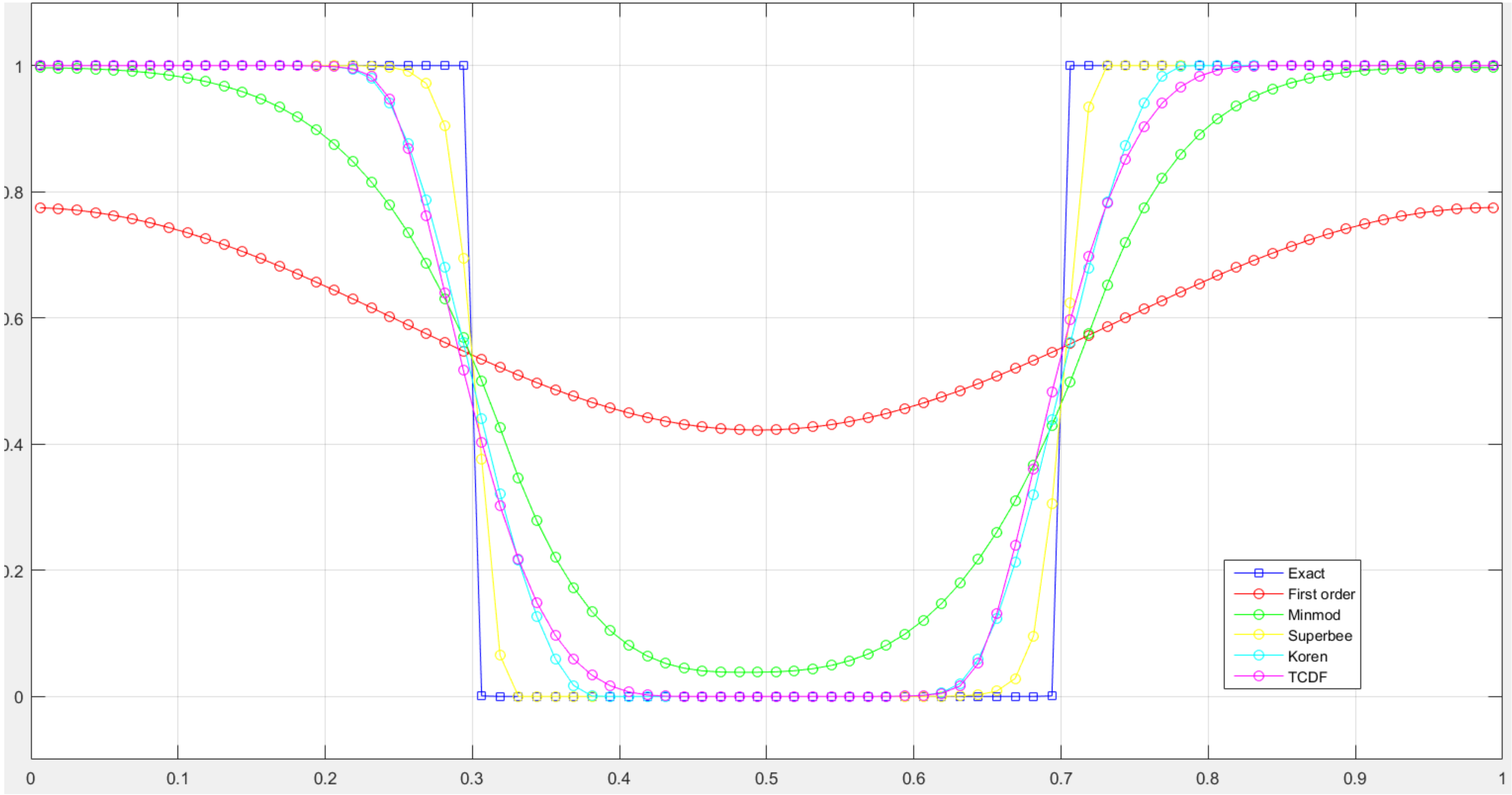


Riemann solver

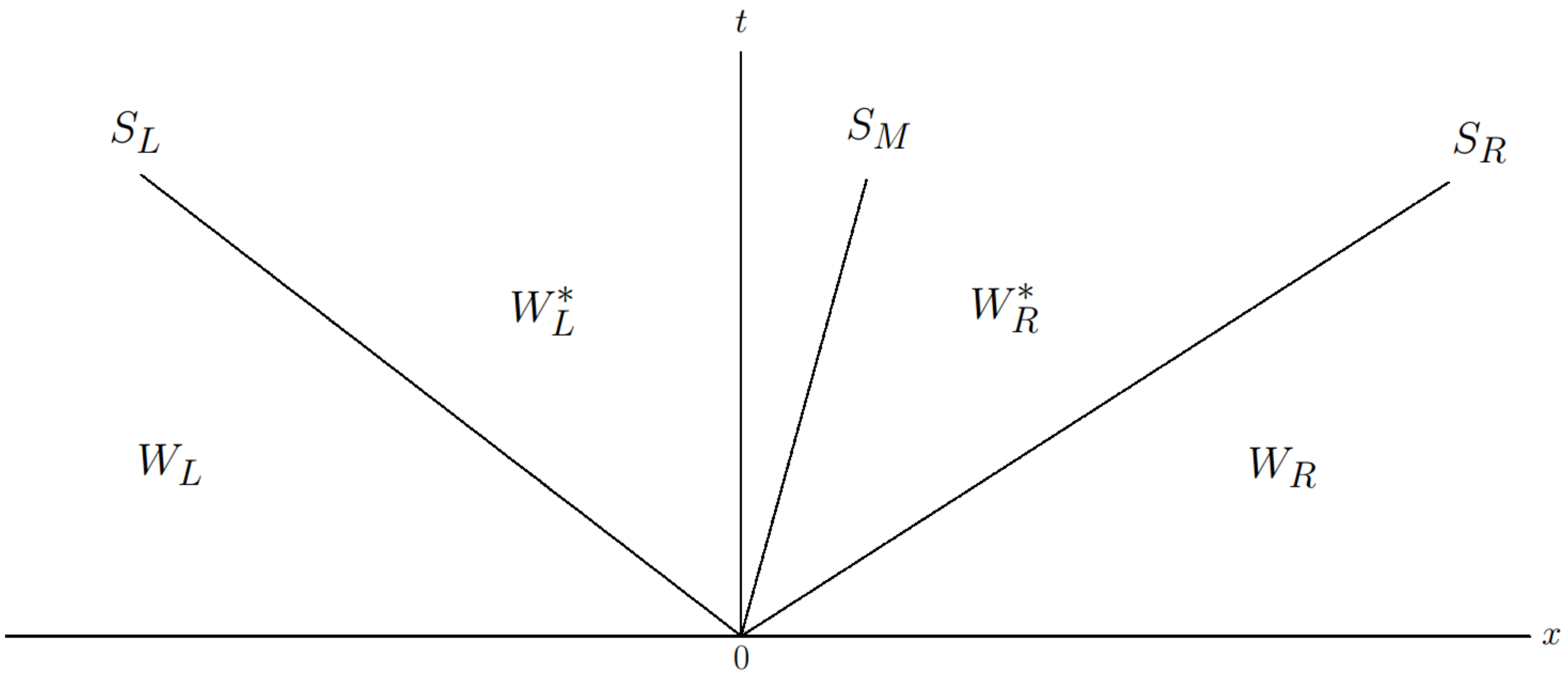
HLLC approx. Riemann solver

Wave speed estimates: S_L, S_M, S_R

Constant intermediate states

Positivity preserving

Shock waves are resolved exactly



Non-conservative terms

$$(\alpha)_t + (\alpha u)_x = (\alpha - \varphi)(u)_x,$$

HLLC type way

$$\Delta t \boldsymbol{S}^n_{x,i,j} = (\alpha_{i,j} - \varphi_{i,j})(u^*_{i+\frac{1}{2},j} - u^*_{i-\frac{1}{2},j})_x$$

u^* taken as the HLLC advection speed

$$u^* = \begin{cases} u_L, & S_L, S_M, S_R > 0 \\ \frac{S_L - u_L}{S_L - S_M} S_M, & S_R, S_M > 0, S_L < 0 \\ \frac{S_R - u_R}{S_R - S_M} S_M, & S_R > 0, S_L, S_M < 0 \\ u_R, & S_L, S_M, S_R < 0 \end{cases}$$

E. Johnsen, T. Colonius, Implementation of WENO schemes in compressible multicomponent flow problems, Journal of Computational Physics, (2006).

Time integration

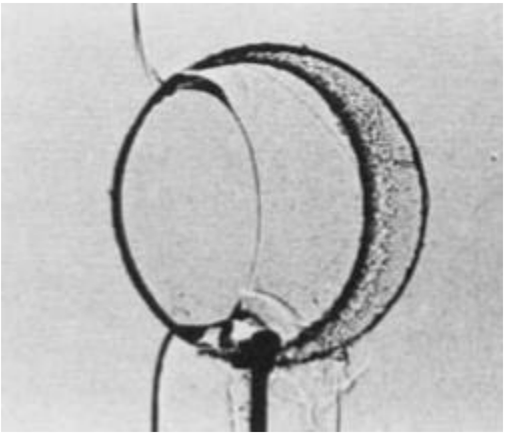
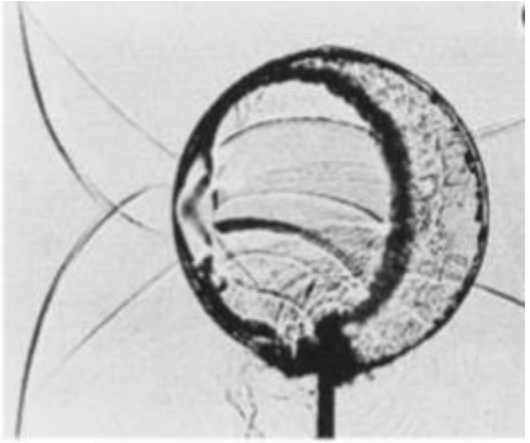
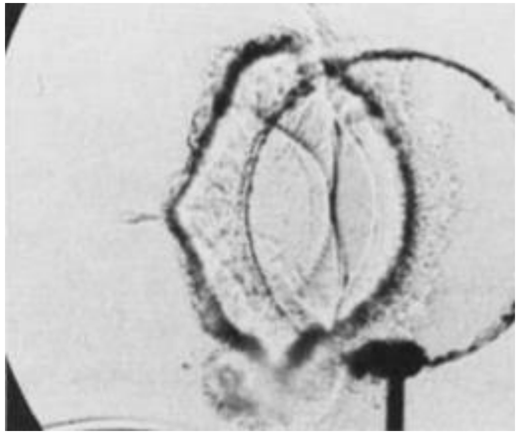
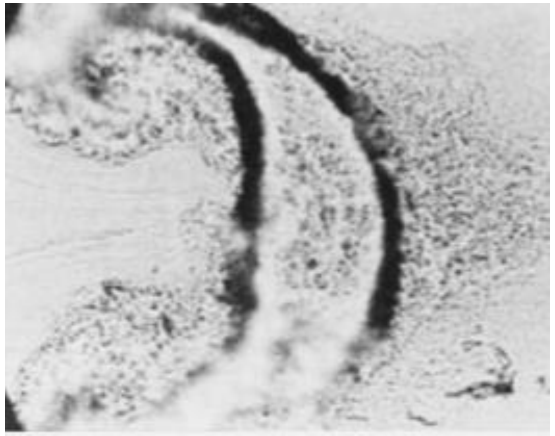
TVD 3rd order Runge Kutta explicit time integration

Time step: CFL condition:

$$\Delta t \leq \frac{1}{4}\min(\Delta x/u_{max}, \Delta y/v_{max,})$$

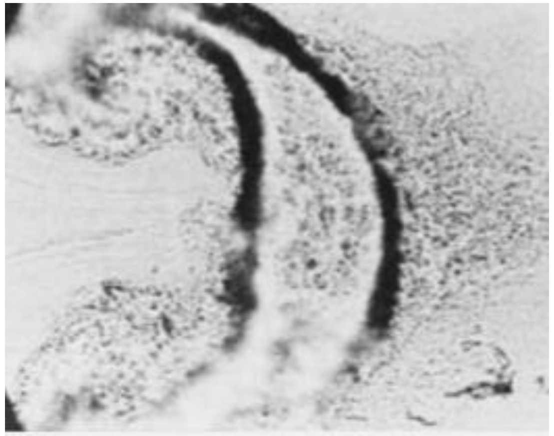
C. Shu, S, Osher, Efficient implementation of essentially non-oscillatory shock-capturing schemes, Journal of computational physics (1988)

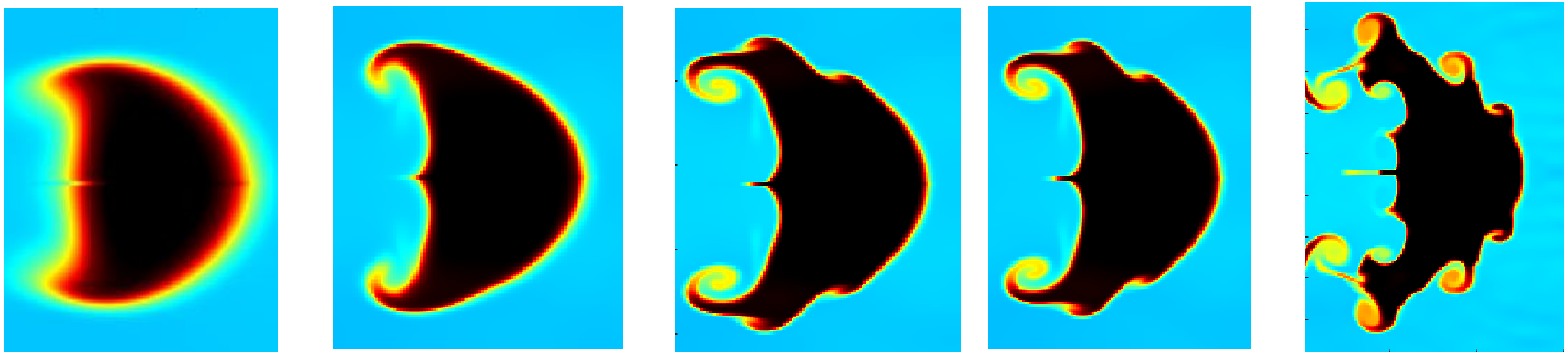
Haas and Sturtevant (1)

J. Haas, B. Sturtevant, Interaction of weak shock waves with cylindrical and spherical gas inhomogeneities, Journal of fluid mechanics, 181, 41-76, (1987)

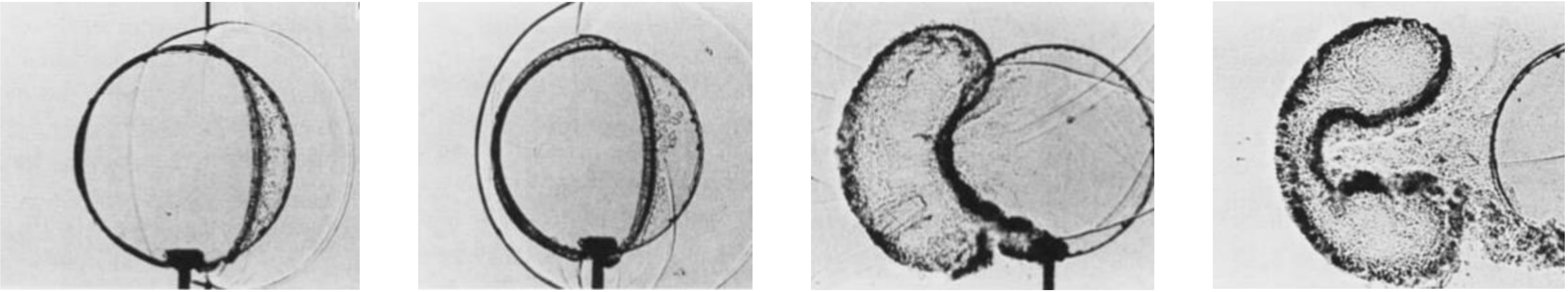
Haas and Sturtevant (1)



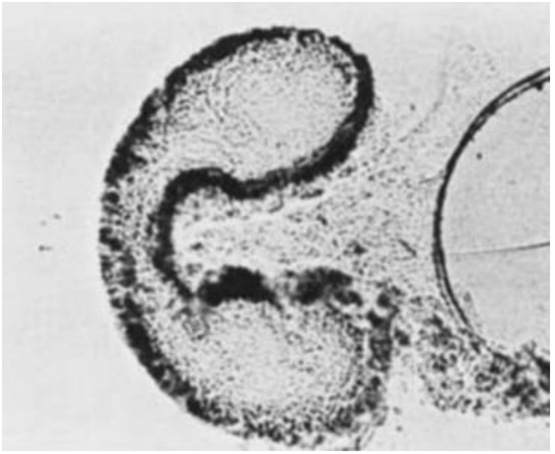


First order | Minmod | Koren | TCDF | Superbee

Haas and Sturtevant (2)



Haas and Sturtevant (2)



First order Minmod Koren TCDF Superbee

Spatial reconstruction

Theory well developed for simple cases

But in this case

- 2D
- System of equations
- Non-linear equations
- Source terms
- 3rd order time integration